MULTI-OBJECTIVE COMPUTATIONAL FLUID DYNAMICS (CFD) DESIGN OPTIMISATION IN COMMERCIAL BREAD-BAKING

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ABSTRACT

Changing legislation and rising energy costs are bringing the need for more efficient baking processes into much sharper focus. High-speed air impingement bread-baking ovens are complex systems used to entrain thermal air flow. In this paper, Computational Fluid Dynamics (CFD) is combined with a multi-objective optimization framework to develop a tool for the rapid generation of forced convection oven designs. A design parameterization of a three-dimensional generic oven model is carried out to enable optimization, for a wide range of oven sizes and flow conditions, to be performed subject to appropriate objective functions measuring desirable features such as temperature uniformity throughout the oven, energy efficiency and manufacturability. Optimal Latin Hypercubes for surrogate model building and model validation points are constructed using a permutation genetic algorithm and design points are evaluated using CFD. Surrogate models are built using a Moving Least Squares approach. A series of optimizations for various oven sizes and flow conditions are performed using a genetic algorithm with responses calculated from the surrogates. This approach results in a set of optimized designs, from which appropriate oven designs for a wide range of specific applications can be inferred. Results from various oven design and objective functions under investigation are presented together with ensuing energy usage and savings. Analysis suggests that 10\% energy savings can be achieved for the baking process.
INTRODUCTION

Commercial bread baking is a complex process of simultaneous heat, water and water vapor transport within the dough/bread in which heat is supplied by a wide variety of indirect-fired and direct-fired forced convection ovens. The former, which rely on radiation from heated elements within the oven, have traditionally been used in commercial bread baking, however the latter have recently been increasing in popularity since they can also offer greater levels of thermal efficiency (Figure 1).

![Figure 1: Schematic diagram of a production oven: (a) Overall view and (b) Cross section view.](image)

Energy use in the process industries is currently an area where a significant amount of research is being conducted. A typical oven may bake between 1000 and 3000 loaves per hour, per m width. Previously, energy use for the production of bread has been estimated to be in the region of between 5 and 10 MJ/kg and typically half of the energy use in a bakery is consumed in the baking oven (Thuman and Mehta, 2008).

Rising energy costs and changing legislation are bringing the need for more efficient baking processes into much sharper focus. This will require greater scientific understanding of how to manipulate oven design and baking conditions to give energy efficient designs whilst maintaining the quality of the product. CFD is increasingly being used to improve the efficiency of baking processes. The capability of CFD modeling to predict airflows and temperature distribution for many applications in the food industry is demonstrated by many authors (Therdthai et al., 2003, Norton and Sun, 2006, Chanwal et al, 2011, Khatir et al,.2011a).

Since the uniformity of the temperature distribution is an important design goal in a baking oven in order to ensure predictable product properties, this will be the primary focus of the present work (Khatir et al., 2010; Therdthai et al., 2002)). In this paper we specifically combine high fidelity CFD analysis and experimentation within a formal optimization framework in order to improve oven efficiency, in terms of: nozzle jets diameter $D$, ratio $H/D$ (i.e. where $H$ is the distance below the nozzle from the impinged surface) and nozzle jets velocity $u_{noz}$, by optimizing oven geometrical and operating parameters. Various objective functions embedding operating conditions as well as energy usage are proposed and formulated within the optimization framework. Results of the various optimization studies are presented and discussed. The newly developed CFD optimization methodology is then used to evaluate the thermal air flow and energy efficiency within commercial bread baking. It is estimated that up to 10% energy savings for the baking process can be achieved.

PROBLEM FORMULATION

Computational Fluid Dynamics

Following Khatir et al. (2011b), air flows in the oven are analysed using the steady-state Navier-Stokes equations for 3D flow. Second order upwind schemes are used for all flow variables and solutions are computed using the SIMPLE algorithm (Patankar and Spalding, 1972). Turbulence is modeled using the realizable $k$-$\epsilon$ transport model as in Khatir et al. (2011b) and Boulet et al. (2010). The continuity, momentum and turbulence transport equations are solved computationally using ANSYS FLUENT 13.0.
Due to geometric and physical symmetry, only the flow field within the solution domain characterized in Fig. 2 together with the boundary conditions is solved numerically.

The geometry shown in Fig. 2b) is composed of flow openings, symmetry planes and walls. For flow openings a combination of velocity inlet and pressure outlet types are used, with temperature and convective heat transfer defined along the walls (see Table 3).

<table>
<thead>
<tr>
<th>Modelled equation</th>
<th>Inlet</th>
<th>Outlet</th>
<th>Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>$T=513$ K</td>
<td>$T=513$ K</td>
<td>(Top) $T = 513$ K (Bottom) $h_c = 10$ W/(m$^2$K)</td>
</tr>
<tr>
<td>Momentum</td>
<td>$V_{in} = u_{in}$</td>
<td>Gauge pressure</td>
<td>$P = 0$ Pa</td>
</tr>
<tr>
<td>Turbulence</td>
<td>$l_{scale} = 5 \times 10^{-4}$</td>
<td>$l_{scale} = 5 \times 10^{-4}$</td>
<td>No-slip Wall function</td>
</tr>
<tr>
<td>Intensity</td>
<td>$I = 2%$</td>
<td>$I = 2%$</td>
<td></td>
</tr>
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</table>

Grid independency analysis is undertaken for both grid cell numbers and grid distributions where values of the temperature uniformity functional $\sigma_T$ defined as

$$\sigma_T = \sqrt{\frac{\int_V (T - T_{zone})^2 dV}{\int_V dV}}$$

are analyzed, where $V$ is the volume of the baking domain, $T_{zone}$ the air-jet temperature in the baking chamber set at 513 K. Results for $H/D=6$ and $D=14$mm are outlined in Fig. 3. Grid independence is achieved with around 0.6 million cells.
Optimisation Strategy

The oven is parameterized by three geometric, \(DV_1 = D\) and \(DV_2 = H/D\) and operating design variables \(DV_3 = u_{noz}\) as indicated in Fig. 4, with a fixed nozzle to nozzle distance spacing \(S = 200\text{mm}\). A surrogate modelling approach is adopted for the optimization study. Such approaches have been successfully used for the design optimization of jet pumps (Fan et al. 2011, Eves et al. 2010). Design of experiments (DoE) is carried out using Optimal Latin Hypercube containing build and validation points. This is achieved via a permutation generic algorithm (Narayanan et al. 2007), applied to the multi-objective problem of optimizing the uniformity of model building, model validation, and combined DoE points. The optimality criterion for each DoE is defined by the Audze-Eglais method (Torooplov et al. 2007) shown in Eq. (2) with the objective function defined by Eq. (3):

\[
U = \sum_{i=1}^{P} \sum_{j=i+1}^{P} \frac{1}{L_{ij}} \tag{2}
\]

\[
F = W_b U_b + W_v U_v + W_m U_m \tag{3}
\]

where \(U\) is a pseudo-potential energy of DOE points, \(L_{ij}\) is the distance between points \(i\) and \(j\) where \(i \neq j\), \(F\) is the objective function to be minimized, \(W\) are weighting factors, and, \(b, v, m\) denote model building, model validation and merged DOEs respectively.

Surrogate models were built using a Moving-Least-Squares (MLS) method where the weighting of points in the regression coefficients calculation are determined using a Gaussian decay function:

\[
w_i = \exp(-\theta \cdot r_i^2). \tag{4}\]

Figure 3: Grid independency: Temperature functional, \(\sigma_T\) (Eq.4) vs. grid size.

Grid Size (Million)

Figure 4: Generic model of oven with design variables: nozzle jet diameter \(D\), jet velocity \(u_{noz}\) and distance \(H\) between bottom impinged surface and nozzle jet, with nozzle to nozzle
A 30 point Optimal Latin Hypercube DOE is constructed with three dimensions using the approach described earlier. Of the 30 points, 20 are building points and 10 are validation points. Equal weights are used in Eq. (2). The levels of the Latin Hypercube are then scaled to correspond to the ranges: $5\text{mm} \leq DV_1 \leq 20\text{mm}$; $2 \leq DV_2 \leq 10$; and $8\text{m/s} \leq DV_3 \leq 40\text{m/s}$. The distribution of points in normalized design variable space is shown in Fig. 5. Values of standard deviation of minimum distances $\sigma_b$, $\sigma_v$ and $\sigma_m$ for building, validation and merged DOEs are found to be $\sigma_b=0.87$, $\sigma_v=1.12$ and $\sigma_m=1.02$ respectively. Figure 6 confirms the uniform scattering of the cloud of design points.
An initial CFD mesh is morphed to match each set of design variables. This is achieved by altering the location of boundary nodes of the bottom plate, as well as the rounded nozzle jet diameter as indicated in Figs. 2 and 4 via a script using GAMBIT mesh software. Mesh checks and smoothing ensure mesh quality is maintained. CFD analysis is performed at each design point using the approach outlined above.

Of primary importance in bread-baking is ensuring good temperature uniformity within the baking chamber to ensure high quality bread. Such a uniformity assessment can be measured via the temperature functional $\sigma_T$ as defined by Eq.(1). The temperature functional $\sigma_T$ values are extracted from the CFD data. MLS approximations of the response is then constructed, using a second order base polynomial and the 20 model building points. The closeness of fit parameter in Eq. (4) is optimized using the 10 model validation points. MLS surfaces gave equally good agreement with building and combined DOEs ($R^2$ values of 0.92 and 0.91 for DOE$_b$ and DOE$_m$ respectively). However there was a slight difference in agreement with the validation points ($R^2$ values of 0.81) due to the complexity of the flow field and the use of relatively few design points. However the current level of approximation is satisfactory within the optimisation framework.

RESULTS

Thermal Airflow Analysis

The complexity of the air flow field within such systems is emphasized in Fig. 8 (a) where multiple recirculation and vortical structures are clearly demonstrated by the pathlines coloured by velocity magnitude. This is also seen through contour plot of velocity magnitude on the front surface plane in Fig. 8 (b).

A uniform temperature within the baking chamber is obtained and confirmed by Fig. 8 where various contour plots of temperature are shown. A difference of temperature $\Delta T_s$ of 10°C between the average temperature at the top of the bread and the baking chamber temperature is obtained. This is an important aspect of the baking process as this will allow the bread to cook efficiently (Khatir et al., 2010; Therdthai et al., 2002). Temperature uniformity is further emphasised in the next section where temperature functional $\sigma_T$ is optimised and discussed.
Cfd Optimisation

First, the optimization problem was formulated in order to minimize the objective function $\sigma_T$. A genetic algorithm (GA) was used to find a global minimum with fitness evaluations carried out by the surrogate models. The GA produced a design which, as predicted by the surrogate model, would reduce temperature difference between the top of the bread (i.e. bottom wall plate in our 3D CFD generic model) and the baking chamber temperature. Results were obtained as follows: $D=20\text{mm}$, $H/D=6.82$ and $u_{\text{noz}}=38.12\text{m/s}$ with a $\sigma_T=1.16$ from the surrogate model which $\sigma_T$ surface response is represented in Fig. 9.

![Figure 9: Surface response $\sigma_T$ from surrogate model with no constraint.](image)

We then formulate the optimisation as follows:

\[
\begin{align*}
\text{minimize} & : & \sigma_T \\
\text{subject to:} & & C_1 \times v^{2/3} - C_2 \times v^2 \geq C_3 \quad \text{where} \quad v = u_{\text{noz}} \\
\end{align*}
\]

The constraint given in Eq. (5) represents a simple model of the difference between the desired heat transfer into the bread (modelled by the $C_1 \times v^{2/3}$ term) and the energy provided to the air jets (modelled by $C_2 \times v^2$). For our current case the following is being used: $C_1 = 5.4$, $C_2 = 0.0039$ and $C_3 = 15$ for the GA optimisation study. The optimised results are found to be $D=20\text{mm}$, $H/D=7.75$ and $u_{\text{noz}}=31.68\text{m/s}$ with a $\sigma_T=1.23$ from the surrogate model. Note that the optimized $H/D$ values of 6.82 and 7.75 are within the range of 6-8 that has been proposed by previous studies (Gardon and Akfirat, 1966; Sarkar and Singh, 2004). The corresponding ratio $S/D$ of 10 used of both cases is in comparatively good agreement with the analysis carried out by Attalla and Specht, (2009).

CFD studies were made of the optimized designs (i.e. 1.05 and 1.23 millions cells are used for the no constraint and with constraint’s case respectively). They showed good agreement with the surrogate models with a $\sigma_T=1.22$ and $\sigma_T=1.30$ which are within 5% of the surrogates’ predictions without and with constraint respectively. A summary of the oven performance at stages of the design process is given in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Response from</th>
<th>$\sigma_T$</th>
</tr>
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<tbody>
<tr>
<td><strong>No Constraint</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Best design from DOE</td>
<td>CFD</td>
<td>1.31</td>
</tr>
<tr>
<td>Optimized design after GA</td>
<td>MLS</td>
<td>1.16</td>
</tr>
<tr>
<td>CFD validation from Optimum</td>
<td>CFD</td>
<td>1.22</td>
</tr>
<tr>
<td><strong>With Constraint Eq. (5)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Optimized design after MLS 1.23
GA
CFD validation from CFD 1.30
Optimum

Table 2: Oven performance at stages of the design process.

The use of the CFD model for energy efficiency assessment is considered next.

Energy Efficiency

The density of the dough/bread is assumed to be constant with value $\rho=330$ kg/m$^3$ and thermal diffusivity $\alpha=2.165 \times 10^{-6}$ m$^2$/s (Wong et al., 2007) with a loaf of bread of 0.25m length, 0.10m width and 0.12m height (i.e. mass of loaf bread of about 1kg) and a convective heat transfer coefficient $h_c=10$ W/(m$^2$K) (Zhang & Datta 2006). It is also assumed that bread is cooked when its core temperature reaches 85°C (Purlis 2011). Complex models predicting the baking process have been developed previously by other authors (Zhang & Datta 2006, Bollada 2008, Jefferson et al 2007), however in this work a simplified heat transfer model is used showing the rate of temperature rise within the bread and does not incorporate moisture content and volume change. This helps to illustrate the scope of potential energy savings within the bread-making industry.

Following the analysis for plane wall heat conduction with convection as described by Incropera et al. (2006) the temperature inside of the dough/bread is modelled by the one-dimensional heat equation:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

for $0 \leq x \leq 2L=0.25$m with the following initial conditions, $T(x,0)=30$ °C and boundary conditions, $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$ and $-K \left.\frac{\partial T}{\partial x}\right|_{x=L} = h_c[T(L, t) - T_{\infty}]$ where $T_{\infty} = 513$ K. The approximate solution of Eq. (10) given by Incropera et al. (2006, p.273) is used to compute the cooking time for various values of $\Delta T_s$, difference between the temperature of top of bread and the baking chamber temperature (i.e. $\Delta T_c = T(L, t) - T_{\infty}$). Calculated cooking times for various values $\Delta T_c$ are summarized in Figs. 10 and 11. $\Delta T_c=0$°C represents the surface of the bread being at the same temperature of the oven, and giving the maximum driving force for heat conduction into the bread (initially at a temperature of 30 °C).

Thus, the optimum design would allow the bread to cook in 24 minutes, that is around 2.5 minutes more than the ideal case $\Delta T_s=0$°C and about 3 and 10 minutes less than for $\Delta T_c=20$°C and $\Delta T_c=40$°C respectively. For practical bread-baking applications a value of 10°C for $\Delta T_c$ is usually acceptable. This would lead to a 5-10% reduction in baking time that results in increased plant efficiency for values of $\Delta T_c$ in the region of 15-20°C.

Figure 10: Cooking time vs. core-bread temperature for various values of $\Delta T_s$, difference between the average temperature of top of bread and the baking chamber temperature.
CONCLUSIONS

A CFD optimisation methodology has been developed and applied within the bread-baking industry as thermal management tool, as well as for reducing energy consumption. This consists in combining high fidelity flow analysis and a formal optimization framework to optimize a set of ovens with various geometric and jet velocities configurations. CFD optimization using a genetic algorithm is performed with the aim of reducing the effective baking time of the bread, and hence improve the energy efficiency of the process. It is shown that 10% energy savings can be achieved for the baking process by reducing the time to bake the bread. Effectively ensuring a uniform temperature within the baking chamber, as well as allowing a temperature differential $\Delta T_s$ between the surface of the bread/dough and the baking chamber of 10°C is financially more effective than $\Delta T_s$ in the region of 15-10°C. Thus, the use of CFD within an optimisation framework, where a suitable objective function is chosen to represent the desired outcome, allows efficient use of computational resource. The choice of objective function is crucial in determining what constitutes an optimal design; the challenge is to interpret the physical parameters (velocities, temperatures, etc) from the CFD solution in a way that links with the objective function of choice. Work is still ongoing and focuses on developing a range of objective functions for energy efficiency, cost or manufacturability within the break baking oven.

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NOMENCLATURE

Greek Symbols
- $\alpha$ Degree of starch gelatinization, [-].
- $\alpha_k$ Thermal diffusivity, $K/(\rho \cdot c_p)$, [m$^2$/s].
- $\Delta T_s$ Temperature difference between the top of bread average temperature and baking chamber, [K].
- $\rho$ Mass density, [kg/m$^3$].
- $\nu$ Kinematic viscosity, [m$^2$/s].
- $\mu$ Dynamic viscosity, [kg/m.s].
- $\sigma_T$ Temperature functional for minimization, [K].
- $\sigma_i$ Standard deviation of minimum distance of DOE$_i$
- $\theta$ Closeness-of-fit parameter.

Latin Symbols
- $Bi$ Biot number, $(h c L)/K$, [-].
- $c_p$ Specific heat, [J/(kg K)].
- $D$ Nozzle jet diameter, [m].
- $DOE_i$ Design of experiment $i$, $i=b, m, v$. 

Figure 11: Cooking time vs. $\Delta T_s$ for the core-bread to reach a temperature of 85°C initially at a temperature of 30°C.
$DV_i$ Design variable $i=1, 2, 3.$
$F$ Objective function.
$g$ Acceleration due to gravity, [m/s²].
$H/D$ Dimensionless nozzle-to-surface distance, [-].
$h_i$ Convective heat transfer coefficient, [W/(m²·K)].
$L$ Characteristic length, [m].
$K$ Thermal conductivity, [W/(m·K)].
$K_i$ Reaction rate constant.
$L_{ij}$ Distance between points $i$ and $j$ in Latin Hypercube.
$Nu$ Nusselt number, $(h_iD)/K$, [-].
$p$ Pressure, [Pa].
$R$ Correlation coefficient.
$Re$ Reynolds number, $(u_{inj}D)/v$, [-].
$r_i$ Normalized distance from the current point to model building point $i$.
$S$ Nozzle to nozzle spacing, [m].
$t$ Time, [s].
$U$ Velocity, [m/s].
$u_i$ Velocity components in the $i$th coordinate direction $x_i$, [m/s].
$U$ Pseudo-potential energy of DoE points.
$w_i$ Weighting factor of the DoE build point $i$.
$x_i$ $i$th coordinate direction, [m].
$u_{inj}$ Nozzle velocity, [m/s].
$V$ Volume, [m³].

Sub/superscripts
$i$ Index $i$.
$j$ Index $j$.
$B$ Model building DOE.
$m$ Combined model DOE.
$\nu$ Model validation DOE.

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