When is a Soliton of Ultracold Matter Stable?

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Figure 1: Above, a bright soliton trapped in a cylindrical "waveguide". This trap essentially acts like a canal for the quantum gas.
Right, conventional waves, like ripples on a pond, spread out over time

Solitons

Conventional waves, like ripples in a pond, spread out over time. Solitons, however, are remarkable one-dimentional waves which maintain their shape and height over large distances. Solitons were first reported in a Scottish canal in 1834. These waves have since been observed in systems as diverse as optical fibres, blood circulation and traffic flow. Most recently, solitons have been formed in ultracold quantum gases. These gases are millions of times less dilute than the gas in a room, are only a few thousandths of a degree above absolute zero, and are roughly the size of a human hair.

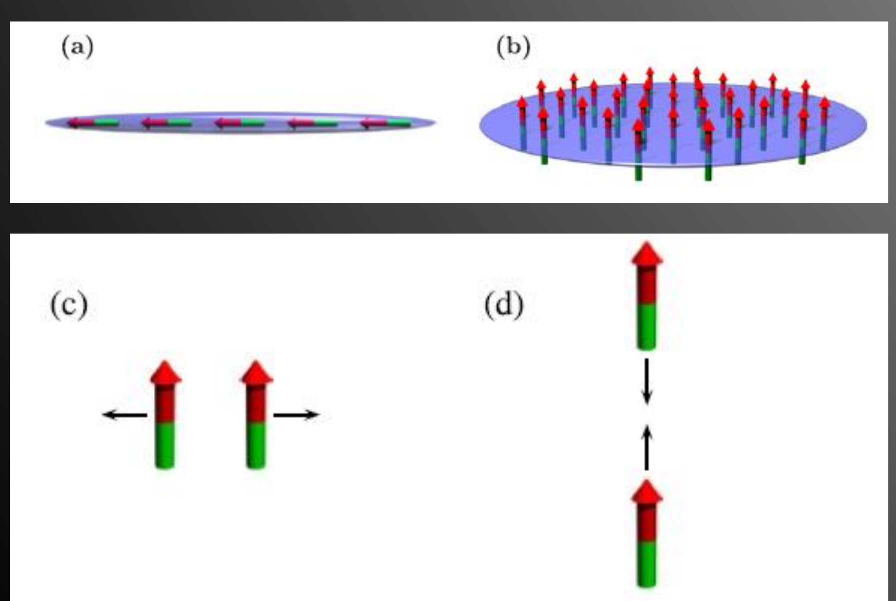
A soliton has 3 key features: [1]
1. They have a permanent, unchanging form,
2. They are localised in space, and,
3. They emerge unscathed from collisions with other solitons.

Depending on whether interactions between particles in the gas are attractive or repulsive, we can form either "dark solitons" or "bright solitons". In this project we will only consider bright solitons. These are formed when the interactions between the particles are attractive. This attractiveness leads to the formation of a dense cloud of matter in the condensate.

Due to the ultracold nature of these gases, we can assume in traditional quantum gases that the molecular van de Waals interactions are the only forces acting on the atoms in this condensate. These forces only happen over an extremely short range, and so the individual atoms only feel this force as they are nearly on top of each other.

Magnetic Interactions Between Particles

Quantum gases have recently been formed where each particle behaves like a magnet. This means that interactions between the particles feature an additional magnetic contribution, known as the dipole-dipole interaction.
A dipole has a pair of equal and oppositely charged poles separated by a distance. The simplest example of a dipole is a bar magnet. These magnets have a north and south pole on opposite ends of a bar. The most important characteristic of dipole-dipole interactions is that they happen over a long range, unlike the particle interactions that we traditionally see in ultracold quantum gases.
The dipole-dipole interactions are made more complicated as the strength and sign of the attraction depends on the angle between the relative positions of the dipoles.
We can also have anti-dipoles; that is, dipoles which behave in the opposite way to a bar magnet. Regular dipoles will attract in a head-to-tail configuration and repel whilst side-by-side. Anti-dipoles repel in a head-to-tail configuration but attract whilst side-by-side.



← Figure 2.
Particles undergoing dipole-dipole interactions.
(a) Dipoles are attractive in a head-to-tail configuration. This makes the cloud long and thin. (b) Anti-dipoles are attractive in a side-by-side configuration making the cloud round and fat.
(c) Two magnets side-by-side would repel ;
(d) Two magnets in a head-to-tail configuration attract.
Diagrams taken from [2].

Aim of the Project

In this project we want to determine the values of molecular and dipolar interactions for which a bright soliton can be formed in a quantum gas.
To do this we minimise the energy functional. This minimum gives us the energetically most favourable configuration of the system.

The Energy Functional

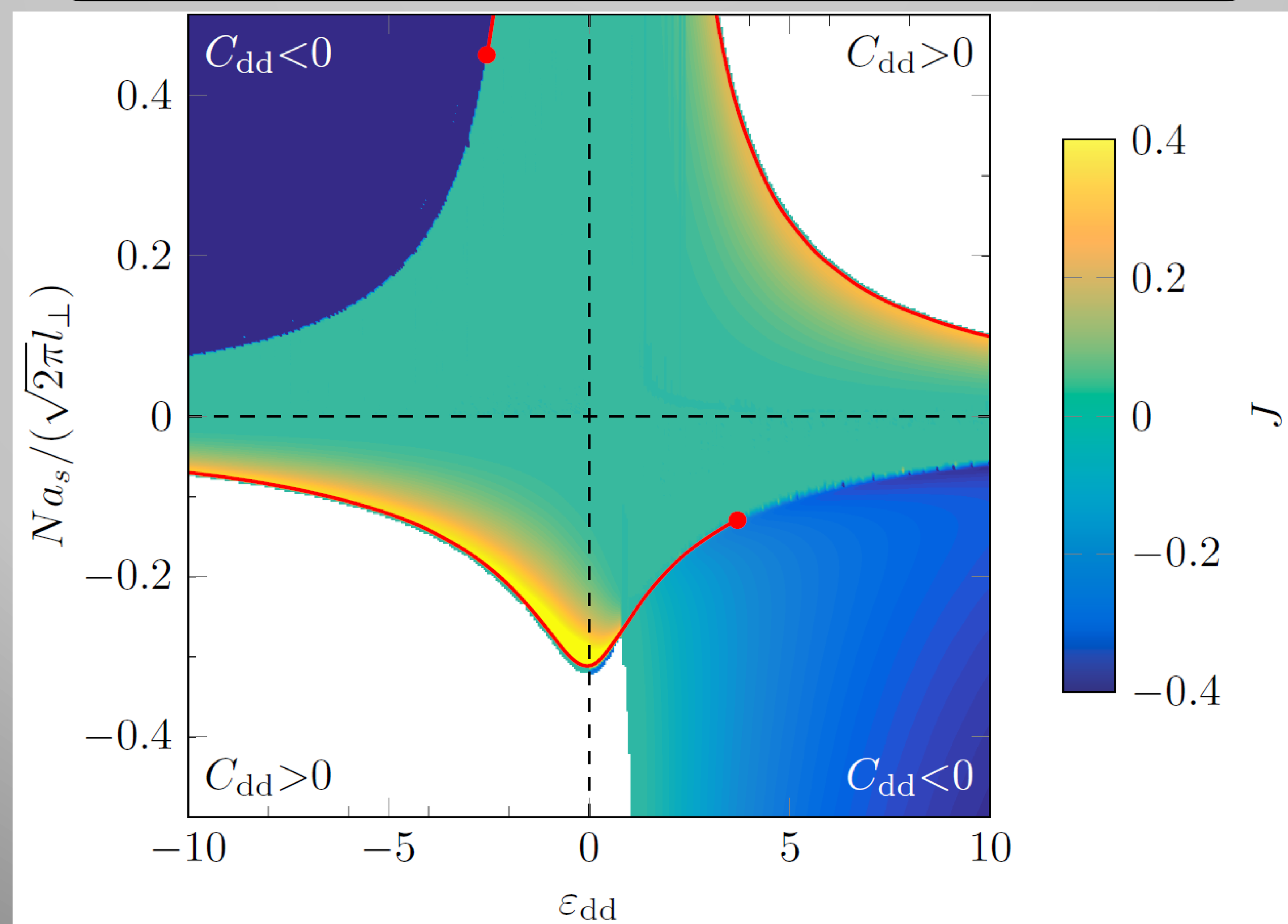
Bright solitons are hump-like blobs of atoms (see Fig. 1). These lumps are formed by a delicate balance between the attractive interactions (required to hold it together), and the natural tendency of a wave to expand. If the interactions are too weak then the wave expands, while if the interactions are too strong then the wave will collapse (destroying it).

There are four parameters which an experimentalist can control:
1. γ: the angle between the z-axis and the direction of polarization of the dipoles
2. λ: the aspect ratio of the trap
3. β: the strength of the interaction between the particles. For $\beta > 0$ we have repulsive particle interactions, and for $\beta < 0$ we have attractive particle interactions.
4. ε_{dd}: the dipole-dipole interaction strength. For $\varepsilon_{dd} > 0$ we have dipole interactions but $\varepsilon_{dd} < 0$ signifies anti-dipole interactions.

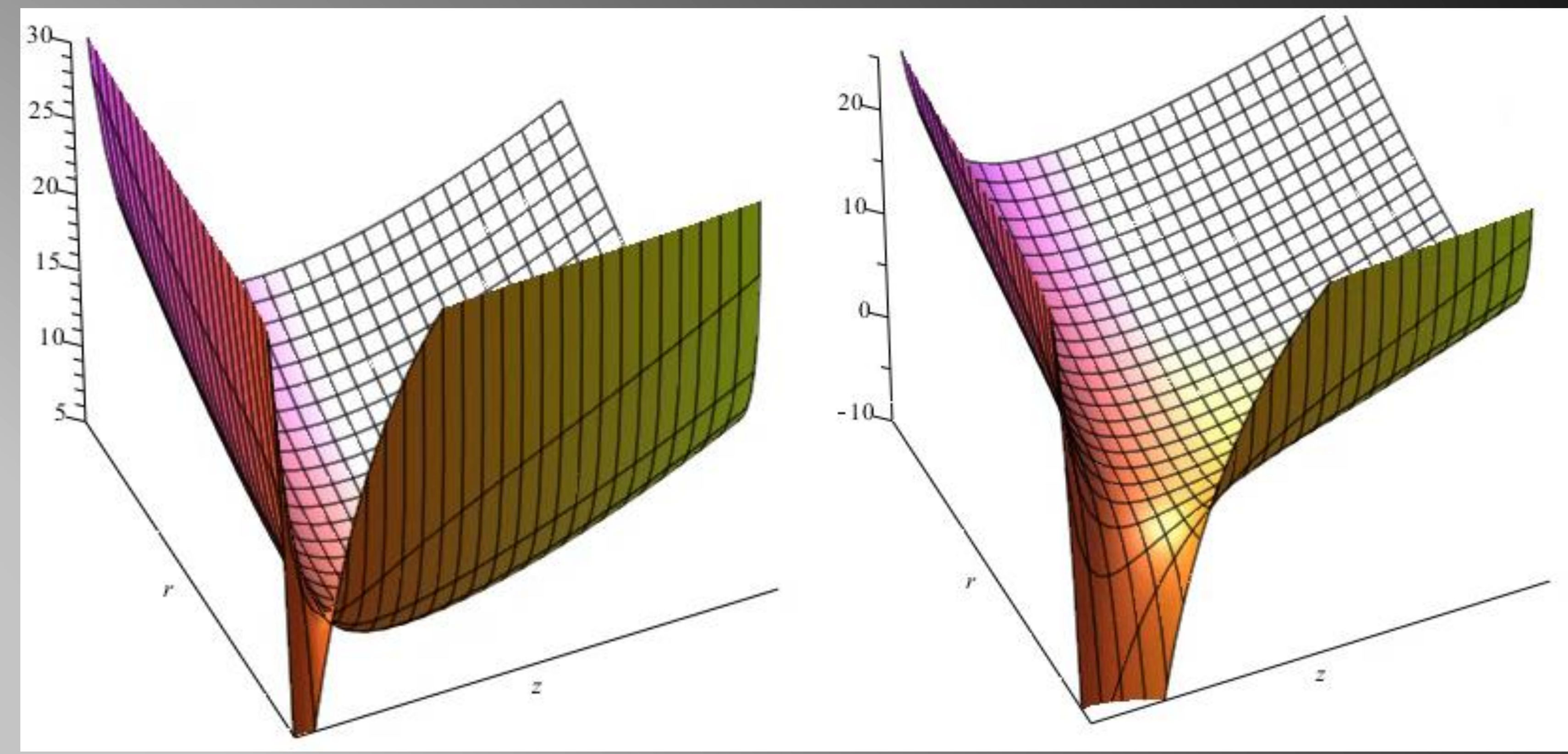
The energy of the soliton depends on these parameters as well as the axial length of the condensate, z, the radial width of the condensate, r, and the aspect ratio of the cloud κ. In terms of these lengthscales, we can calculate the energy of the soliton

$$E[r,z] = \frac{1}{4}\left(\frac{1}{z^2}+\frac{2}{r^2}\right)+\frac{r^2}{2}+\frac{\lambda^2}{2}z^2 + \frac{\beta}{2r^2 z}\left[1+\frac{\varepsilon_{dd}(1-3\cos^2\gamma)}{2}\left(\frac{1+2\kappa^2}{1-\kappa^2}-\frac{3\kappa^2\,\mathrm{arctanh}\sqrt{1-\kappa^2}}{(1-\kappa^2)^{\frac{3}{2}}}\right)\right].$$

Like a ball falling back to earth or water running to the lowest point in a bath, the condensate will always try to minimise the energy of the system. Using the mathematical formulation of the energy above, we can find this minimum in terms of the lengths of the soliton. On finding this minimum, we wish to examine its stability.



↑Figure 3.
A plot showing the boundaries of the region which supports a stable soliton in $\varepsilon_{dd} - \beta$ parameter space.
Here for $J > 0$ the system supports the formation of a bright soliton. The blue areas, $J < 0$, represent a runaway collapse of any soliton and the white regions denote the areas of parameter space where a soliton would collapse into a point.
Figure taken from [3].



↑ Figure 4. Landscapes similar to those formed by the energy functional.
Left: A landscape which supports the formation of a soliton. There is a bowl which is a local minima on the energy landscape. There is also a barrier between this bowl and the global minima at the origin which prevents a collapse.
Right: There is no barrier to prevent a collapse. The energetically most favourable configuration in this situation is for the condensate to collapse into a point. This is due to overwhelming attractive forces between the particles.

Stability

The energy landscape where a minima can be found are similar to the surface on the left of Fig. 4. Here there is a bowl shaped structure which is the local minima of the landscape. Importantly, there is a small raised barrier to separate the bowl shaped minima from the chute.

Mathematically, we classify the bowl shaped minima as stable. If we imagine rolling a small marble on the energy landscape on the left of Fig. 4, it is easy to believe that it will settle back into the bowl which is the minima. If, on the other hand, we rolled a marble on the energy landscape on the right of Fig. 4, the marble would run off down the chute which has been formed. This is analogous to the condensate collapsing into a point.

Minimising the energy functional $E[r,z]$, we can find the values of ε_{dd} and β for which we can form a bright soliton. These solutions are displayed in Fig. 4.
- For $J > 0$ (the yellow/green regions), we have the formation of a bright soliton. This is characterised by an energy landscape which looks similar to the surface on the left of Fig 4. The energy minima in this case is stable, preventing the soliton from either collapsing into a point or spreading out.
- For $J < 0$ (the blue regions), the soliton will undergo a runaway expansion. This is because the repulsive interactions dominate, and so the energetically most favourable configuration of the system it to spread out.
- In the white regions of Fig. 3, we can not find a soliton. In these regions, the attractive interactions are dominant and so the condensate will collapse into a point. In this case, the energy functional takes the form of the surface on the right of Fig. 4.

That we can find bright solitons in regimes which have repulsive molecular interactions means that dipolar quantum gases are fundamentally different to their traditional counterparts.

Conclusions

The most interesting finding in this project is that by using a dipolar quantum gas (rather than a traditional one), we can find bright solitons in regions where we have repulsive particle interactions. This is attributed to having a strong enough dipole-dipole attraction that we can overcome the repulsive interactions of the molecular forces.

In this project, we considered dipoles that were polarized along the z-axis. A natural extension of this work would be to repeat the analysis of the energy functional for different angles of polarization.

References

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[2] T. Lahaye et Al. *The physics of dipolar bosonic quantum gases,* Rep. Prog. Phys. **72,** 126401 (2009)
[3] M. Edmonds, T. Bland, R. Doran & N. G. Parker *Engineering Bright Matter-Wave Solitons of Dipolar Condensates (arXiv: 1610.01022),* recently submitted to *New Journal of Physics*