Morphology of random fields: Application to interstellar neutral hydrogen

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Objective: To investigate small scale fluctuations in the density of the hydrogen gas in the Milky Way.

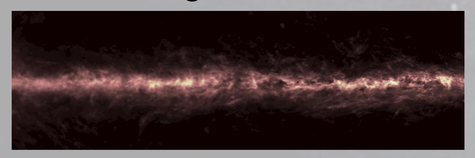
Data

The observational data that we have used comes from the *Parkes Galactic All Sky Survey*. [1] After transforming the data into cylindrical polar coordinates (R, Φ, z) we then split it into slices according to galactocentric radii at $R = 10, 12, 14, \dots, 24, 27$ kiloparsecs.

To extract the small scale fluctuations from the general trend of the galactic disk we used a method called wavelet filtering. [2]

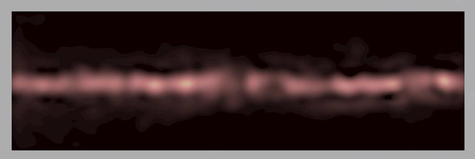
This images below show the result of this separation for $R = 12$:

Original data:



Trend:

Fluctuations:

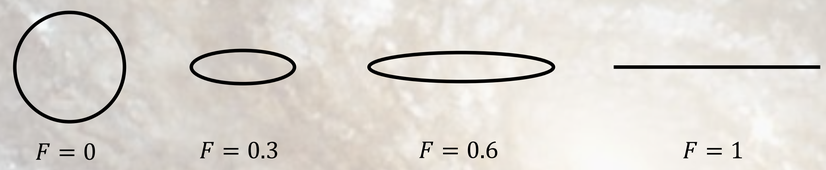


Filamentarity

Filamentarity (F) is a 2D dimensionless measure of how stretched out an object is. It is independent of the object's size and is calculated using the following formula:

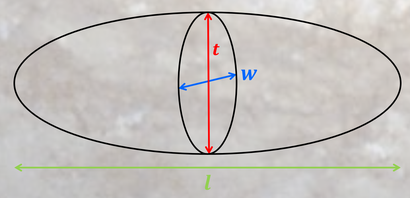
$$F = \frac{P^2 - 4\pi S}{P^2 + 4\pi S},$$

where P is the perimeter of the object, S its enclosed area and $0 \leq F \leq 1$.

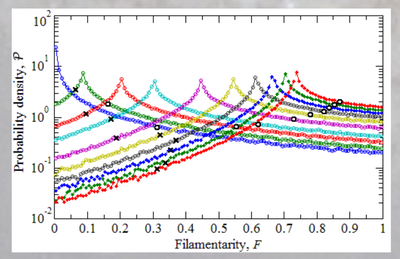


Theory

By plotting PDFs of the filamentarity values for randomly oriented cross-sections of ellipsoids with known aspect ratios, it has been found that the peak of this PDF is determined by the ratio of width to thickness $\left(\frac{w}{t}\right)$ and the value at which it truncates by that of length to thickness $\left(\frac{l}{t}\right)$. [3]



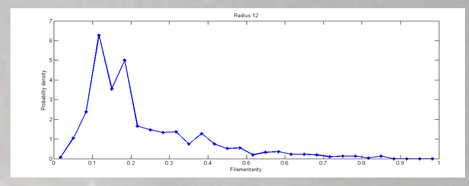
The figure to the right shows a PDF of random cross-sections of infinitely long elliptical cylinders with a range of aspect ratios.



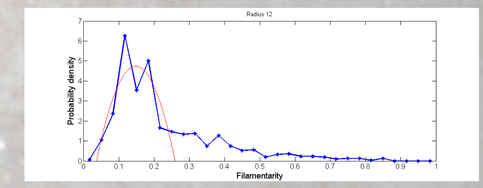
Application and findings

The image below shows the exclusion field for $R = 12$, produced by setting all points below a threshold equal to zero and all points above the threshold equal to 1

By calculating the filamentarity values of the structures within this field the following PDF has been produced.



In order to establish the peak of the PDF a parabolic curve was fitted to the line using least squares regression.



From the graph we can estimate that for $R = 12$, $\frac{w}{t} = 2.437$ and $\frac{l}{t} = 37.084$. The table below shows the aspect ratios of the structures at the remaining radii using the same method.

R	$\frac{w}{t}$	$\frac{l}{t}$
10	2.483	32.404
14	2.405	51.998
16	2.424	56.708
18	2.398	54.717
20	2.347	31.079
22	2.405	27.034
24	2.359	26.399
27	2.263	34.417

References:

[1] Parkes Galactic All-Sky Survey. <url: http://www.atnf.csiro.au/research/GASS/Data.html> [Accessed: 29 October 2014]
[2] Makarenko, I., Fletcher, A., Kalberla, P. M. W. and Shukurov, A. *Neutral hydrogen in the outer Galaxy. I. Large-scale features.* Mon. Not. R. Astron. Soc. (2014, in submission)
[3] Makarenko, I., Fletcher, A. and Shukurov, A. *3D morphology of a random field from its 2D cross-section.* arXiv:1407.4048v1 [physics.flu-dyn] (2014)