Statistical Analysis of Vorticity Transport in Turbine Premixed Flames

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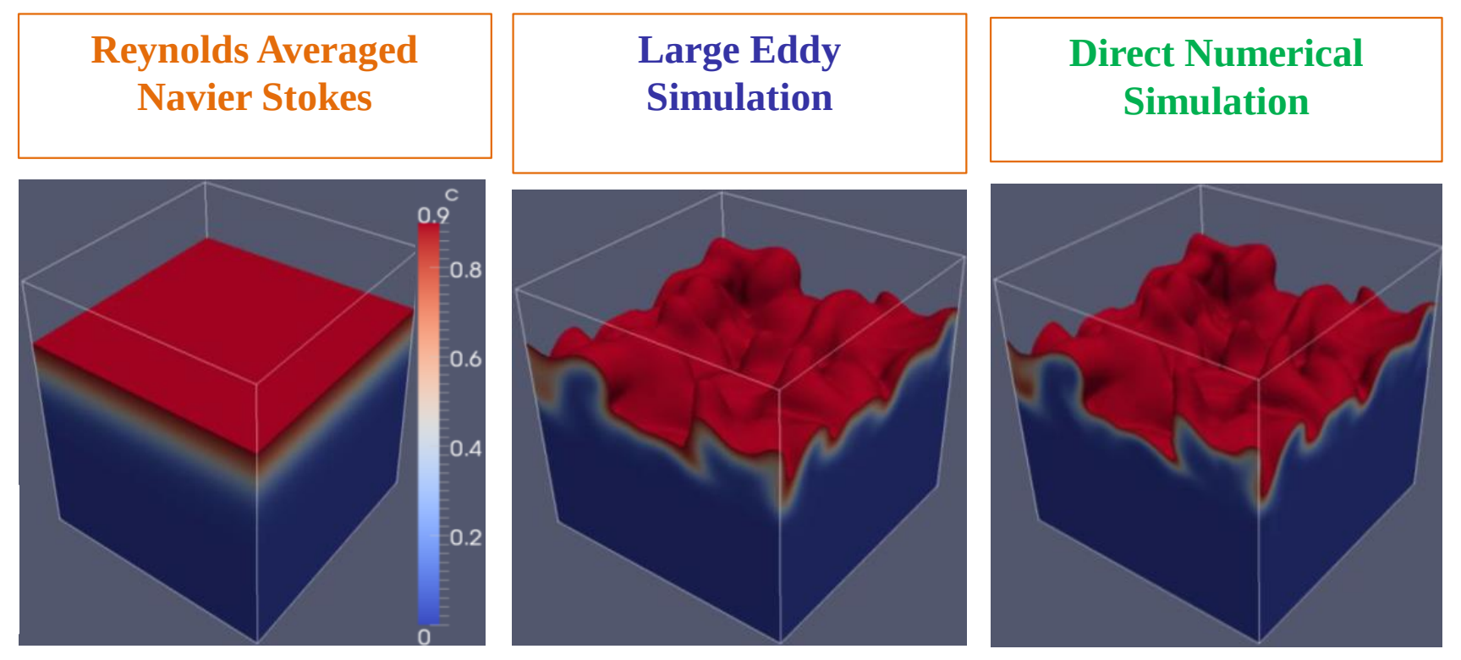
1 Introduction

- The statistical behaviour of vortcity (i.e. curl of velocity) and enstrophy (=0.5*voriticity magnitude squared) transport in the context of Reynolds-Averaged Navier Stokes (*RANS*) simulations has been analysed based on a Direct Numerical Simulation (*DNS*) database [1] of statistically planar flames for a range of Lewis numbers (*Le*) and turbulent Reynolds numbers (Re_t).
- The effects of differential diffusion of heat and mass is characterised by a non-dimensional number known as Lewis number *Le*, which is the ratio of thermal diffusivity to mass diffusivity. Light hydrocarbon-air mixture (e.g. $Le \approx 1.0$ for stiochiometric CH_4-air flame, $Le > 1$ for heavier hydrocarbon-air flames and $Le \ll 1$ for lean H_2-air flames.
- Lewis number affects overall burning rate, flame winkling and has significant influences on flame-generated turbulence in turbulent premixed flames [2]. Thus, *Le* is likely to have significant influences on vorticity transport in premixed flames.

RANS → already widely used in industry

LES → more expensive than RANS; more accurate for large scale unsteadiness; becoming popular in industry

DNS → computationally expensive (CPU cost ~ Re_t^3); highest possible resolution in 3D



2 Mathematical Background

Reynolds-averaged enstrophy transport equation:

$$\underbrace{\overline{\rho\frac{\partial \Omega}{\partial t}}}_{\text{Transient Term}} + \underbrace{\overline{\rho u_j \frac{\partial \Omega}{\partial x_j}}}_{\text{Advection Term}} = \underbrace{\overline{\rho\omega_i\omega_j\frac{\partial u_i}{\partial x_j}}}_{\text{Vortex Stretching Term}} - \underbrace{\overline{\rho\omega_i\omega_i\frac{\partial u_j}{\partial x_j}}}_{\text{Dilatation Term}} + \underbrace{\overline{\varepsilon_{ijk}\frac{\omega_i}{\rho}\frac{\partial\rho}{\partial x_j}\frac{\partial p}{\partial x_k}}}_{\text{Baroclinic Torque Term}}$$

$$- \underbrace{\overline{\varepsilon_{ijk}\frac{\omega_i}{\rho}\frac{\partial\tau_{kl}}{\partial x_l}\frac{\partial\rho}{\partial x_j}}}_{\text{Viscous Torque Contribution}} + \underbrace{\overline{\frac{\partial}{\partial x_j}\left[\mu\frac{\partial\Omega}{\partial x_j}\right]}}_{\text{Diffusion of Vorticity Term}} - \underbrace{\overline{\mu\frac{\partial\omega_i}{\partial x_j}\frac{\partial\omega_i}{\partial x_j}}}_{\text{Dissipation Term}} + \underbrace{\overline{\varepsilon_{ijk}\frac{\mu\omega_i}{3}\frac{\partial^3 u_l}{\partial x_j\partial x_l\partial x_k}}}_{\text{Dilatation – viscous contribution}}$$

where

t	time
x_i	spatial coordinates
$\overline{\Omega}$	Reynolds-averaged (mean) enstrophy
ω_i	i^{th} component of vorticity
u_i	i^{th} component of velocity
p	pressure
μ	dynamic viscosity
ρ	density
τ_{ij}	viscous stress tensor
τ	heat release parameter
n_i	normal vector to the flame brush
S_L	lamivar burning velocity
δ_{th}	flame brush thickness
$\tilde{c}$	Favre-averaged combustion progress variable

The viscous stress tensor is given by:

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right),$$

the Reynolds-averaged enstrophy is given by:

$$\overline{\Omega} = \frac{\overline{\omega_i\omega_i}}{2},$$

and the normal vector to the flame brush is given by:

$$n_i = -\frac{1}{|\nabla c|}\frac{\partial c}{\partial x_i}\bigg|_{c=c^*}$$

3 Numerical Implementation

Simulation Parameters

u'/S_L	l/δ_{th}	Re_t	Da	Ka
5.0	1.67	22	0.33	8.65
6.25	1.44	23.5	0.23	13.0
7.5	2.5	49.0	0.33	13.0
9.0	4.31	100	0.48	13.0
11.3	3.75	110	0.33	19.5

Le	u'/S_L	δ_L/δ_{th}	l/δ_{th}	Re_t	Da	Ka
0.34	7.5	2.17	2.45	47.0	0.33	13.0
0.6	7.5	1.40	2.45	47.0	0.33	13.0
0.8	7.5	1.15	2.45	47.0	0.33	13.0
1.0	7.5	1.0	2.45	47.0	0.33	13.0
1.2	7.5	0.90	2.45	47.0	0.33	13.0

$$\tau = 4.5\,;\ \beta = 6.0\,;\ Pr = 0.7\,;\ Ma = S_L/\sqrt{\gamma R T_0} = 0.014159$$

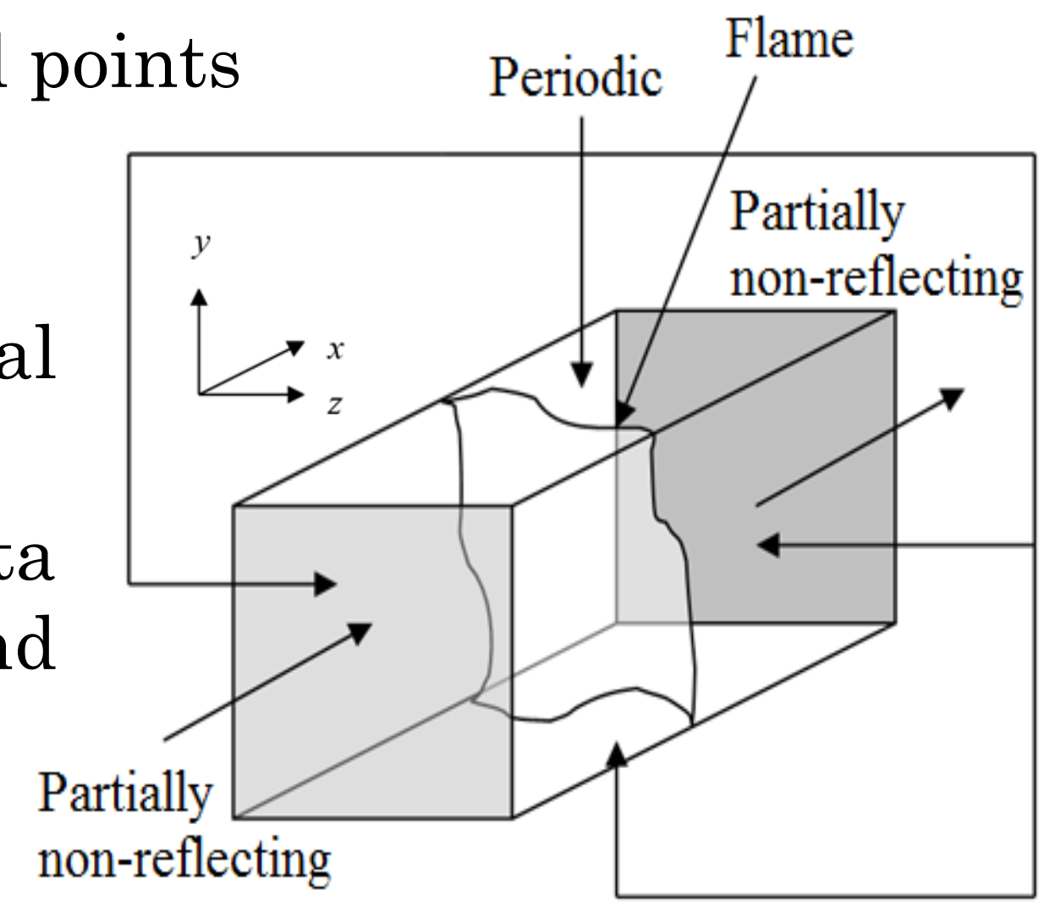
$$Re_t = \frac{\rho_0 u' l}{\mu_0},\quad Le = \frac{\lambda}{\rho C_P D},\quad Pr = \frac{\mu C_P}{\lambda},\quad Sc = \frac{\mu_0}{\rho_0 D},\quad \beta = \frac{E_{ac}(T_{ad}-T_0)}{R T_{ad}{}^2},$$

$$Ma = \frac{S_L}{\alpha_0},\quad Da = \frac{l S_L}{u'\delta_{th}},\quad \tau = \frac{T_{ad}-T_0}{T_0},\quad Ka = \left(\frac{u'}{S_L}\right)^{\frac{3}{2}}\left(\frac{l}{\delta_{th}}\right)^{-\frac{1}{2}},\quad \gamma = \frac{C_{P0}}{C_{V0}}$$

DNS Database

$$\left.\begin{aligned} Le &: 24\delta_{th} \text{ x } 24\delta_{th} \text{ x } 24\delta_{th} \\ Re_t &: 36.2\delta_{th} \text{ x } 24\delta_{th} \text{ x } 24\delta_{th} \end{aligned}\right\} \begin{aligned} &230 \text{ x } 230 \text{ x } 230 \\ &345 \text{ x } 230 \text{ x } 230 \end{aligned} \quad \text{grid points}$$

- Conservation equations of mass, momentum, energy and species are solved in non-dimensional form.
- High order finite difference and Runge-Kutta scheme are used for spatial discretization and explicit time advancement, respectively.



4 Results - I

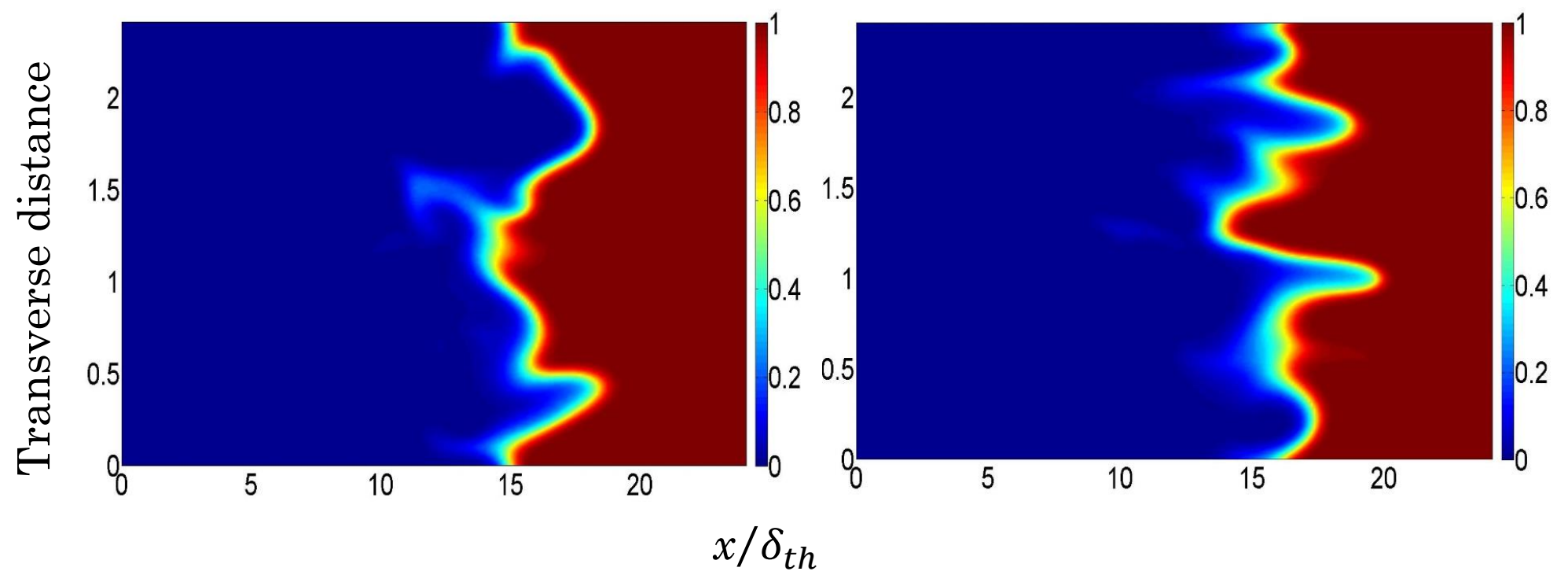
Reaction Progress Variable

$$c = \frac{Y_P - Y_{P0}}{Y_{P\infty} - Y_{P0}}$$

c is 0 in unburned reactants (shown in blue)

c becomes unity in fully burned products (in red)

$\tilde{c}$ along the x-y and x-z planes of the cubic size domain.

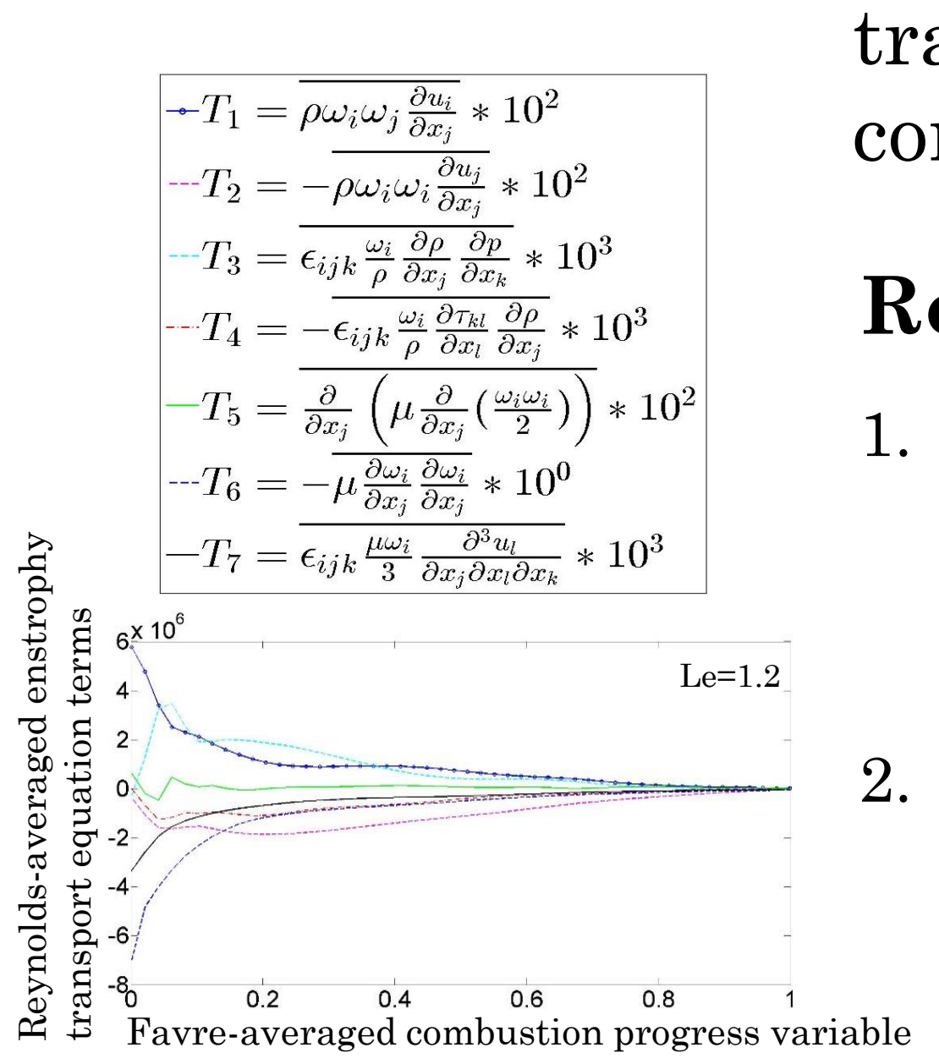


Y_{P0} is the product mass fraction.

Subscripts 0 and infinity are used to refer to the values in unburned gas and completely burned gas respectively.

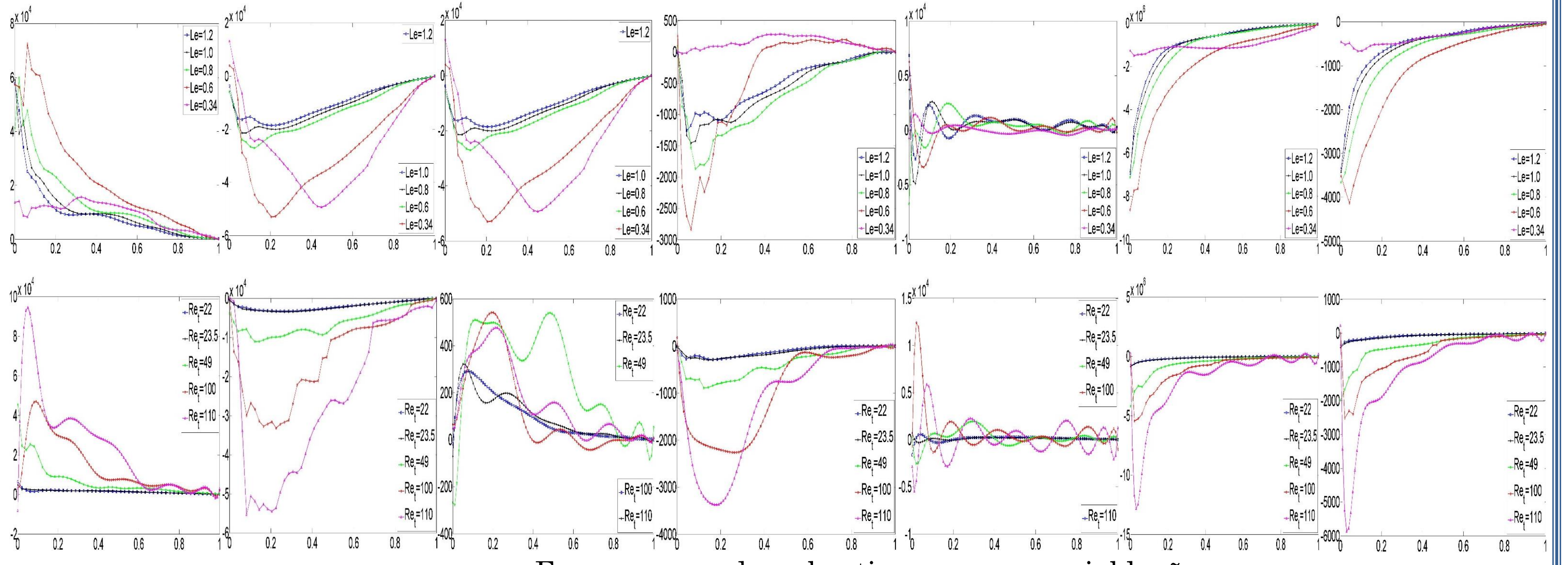
Effects of *Le* and Re_t

- Mean enstrophy increases with decreasing *Le* , whereas the mean enstrophy increases with increasing Re_t.
- The magnitudes of the baroclinic torque term , viscous torque and dilatation-viscous contributions remain smaller than the magnitude of the dissipation term irrespective of the value of *Le*.
- The dissipation term remains the leading order sink irrespective of the value of Re_t (e.g. the order of magnitude of dissipation term is three times greater than the rest of the terms). The role of this term in the system is destructive (i.e. damp the enstrophy), thus its negative sign.

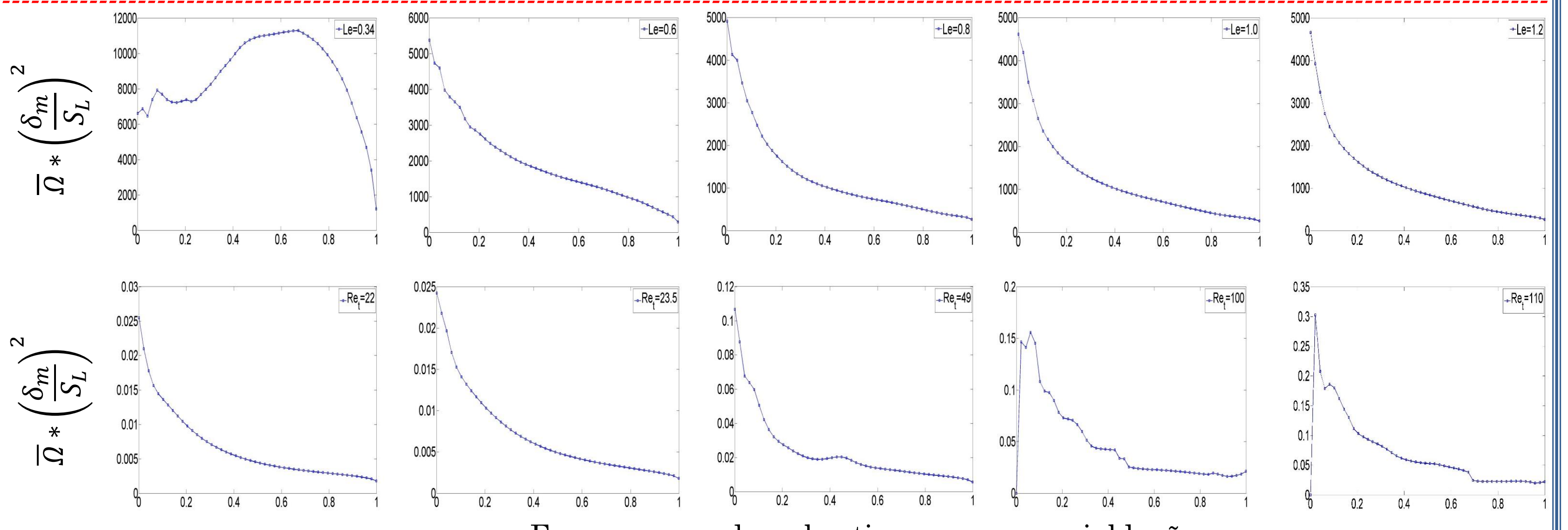


5 Results - II

$$\overline{\rho\omega_i\omega_j\frac{\partial u_i}{\partial x_j}} \quad -\overline{\rho\omega_i\omega_i\frac{\partial u_j}{\partial x_j}} \quad \overline{\varepsilon_{ijk}\frac{\omega_i}{\rho}\frac{\partial\rho}{\partial x_j}\frac{\partial p}{\partial x_k}} \quad -\overline{\varepsilon_{ijk}\frac{\omega_i}{\rho}\frac{\partial\tau_{kl}}{\partial x_l}\frac{\partial\rho}{\partial x_j}} \quad \overline{\frac{\partial}{\partial x_j}\left[\mu\frac{\partial\Omega}{\partial x_j}\right]} \quad -\overline{\mu\frac{\partial\omega_i}{\partial x_j}\frac{\partial\omega_i}{\partial x_j}} \quad \overline{\varepsilon_{ijk}\frac{\mu\omega_i}{3}\frac{\partial^3 u_l}{\partial x_j\partial x_l\partial x_k}}$$



Favre-averaged combustion progress variable, $\tilde{c}$



Favre-averaged combustion progress variable, $\tilde{c}$

6 Conclusions

- The statistical behaviour of enstrophy transport in turbulent premixed flames has been analysed using a three-dimensional DNS database of statistically planar freely propagating turbulent premixed flames with different values of Re_t and *Le*.
- *Le* changes the qualitative variation of enstrophy within the flame.
- The magnitude of the terms of the enstrophy transport equation changes, but the qualitative behaviour remains the same in response to the changes in Re_t for given *Le*.
- The vortex stretching, baroclinic torque, diffusion of vorticity terms and dilatation-viscous contribution act as source terms in the enstrophy transport equation, whereas the dilatation term, viscous torque contribution and dissipation term act as sinks.

References

1. Chakraborty N, Fundamental Study of Turbulent Premixed Combustion using Direct Numerical Simulation, PhD thesis, Chapter 2, Mathematical Background, Cambridge University 2004
2. Chakraborty N, Katragadda M, Cant RS. Effects of Lewis number on turbulent kinetic energy transport in turbulent premixed combustion. Physics of Fluids 2011, 23, 075109.