EXOTIC BLACK HOLES[*]

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Abstract

Black hole solutions can be used to shed light on general issues in General Relativity and Quantum Physics. Black–hole hair, entropy and naked singularities are considered here, along with some implications for the inflationary universe scenario.

1 Introduction

Black holes form as a result of gravitational collapse which may be very variable in nature, but a significant simplification results from the observation that the final state of gravitational collapse seems to be characterised by only a few parameters. This is the phenomenon that John Wheeler described as the loss of hair by the black hole (Ruffini and Wheeler 1971) . It seems to be a feature of the surface or the event horizon that defines the limit of communication with the outside.

Over the last few years a number of new black hole solutions have been discovered. These include solutions that do not lose their hair entirely, they have some residual fields outside of their event horizons and sometimes need extra parameters to distinguish them. By studying these solutions it is possible to understand fundamental issues of black hole physics a little better.

One important issue is that of black hole thermodynamics. For an event horizon of area $\mathcal{A}$ we are able to associate an entropy S,

$$S = \tfrac{1}{4}\mathcal{A} \tag{1}$$

(in units where $\hbar = c = G = k = 1$). It seems unusual to assign an entropy to a classical field configuration, but the justification lies in the extent to which the laws of thermodynamics can be applied to a black hole. The relationship is consistent with the second law of thermodynamics and seems natural on the grounds of information swallowed by the hole.

The discovery made by Stephen Hawking was that thermodynamical equilibrium of the hole and its surroundings could only be achieved by including quantum mechanical pair creation. This allows the hole to radiate as well as absorb energy. The temperature is given by the Hawking formula $T = \kappa/2\pi$, where κ is the surface gravity of the event horizon.

Even this, however, cannot take us fully into the range of thermodynamics because the equilibrium between a black hole and its surroundings would usually be unstable. This is an effect of having a negative specific heat. The black hole forces us to reconsider the basic laws of thermodynamics just as is forces us to question the basis of unitary evolution in quantum mechanics.

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There is also an interesting class of black hole solutions that include a cosmological constant. The loss of hair for these holes is important for the homogeneity of the early universe if it underwent a period of inflation. In fact the existence of these solutions shows explicitly that the universe does not become homogeneous, but it is consistent with the 'no hair' theorems for the geometry to approach de Sitter assymptotically.

Beyond the event horizon, in the interior, lies a region of space that pushes our physical knowledge to its limits. According to general relativity there is a spacelike singularity, or a null singularity if the black hole is rotating. This singularity would define the end of time as far as classical evolution is concerned.

A far more serious condition is encountered if the singularity is timelike. The class of solutions that describe black holes in a closed universe with a cosmological constant do seem to have a singularities of this kind. The time evolution from a singularity is not unique in Einstein gravity and therefore the spacetime becomes unpredictable before time comes to an end. It is even possible for time–like curves to loop back on themselves. These features demand a theory which goes beyond Einstein's theory of gravity, possibly a theory of quantum gravity.

2   No–Hair Theorems

Originally the no–hair theorems arose from attempts to understand the nature of the event horizon that remains after gravitational collapse (Israel 1967). The early work is summarised by Price's theorem (Price 1972):

The only stationary solutions to the massless wave equations with spin $s = 0, 1, 2$ on a spherically symmetrical black hole background have angular momentum less than s.

The implication is that all of the other modes are radiated away during the collapse and that the event horizon is quite easy to characterise. Any field which violates the conclusion of Price's theorem might therefore be described as black hole hair.

There is a more useful view, which can be found in the work of Brandon Carter for example (Carter 1979), that the no–hair theorems are statements about the number of parameters that are needed to specify the equilibrium state. In Einstein–Maxwell theory there are just three of these parameters:

mass M	surface gravity κ
charge Q	potential α
angular momentum J	rotation Ω

In this list the set of quantities that are measured far from the hole are listed on the left, but there is also an alternative set of parameters that characterise the horizon itself listed on the right. These quantities are defined by properties of the various killing fields that are present.

For equilibrium the solutions have a killing field k which is timelike at large distances from the hole but also, because of a theorem due to Hawking (Hawking 1972), a killing field m for rotational symmetry. The horizon is generated by a null combination of the two fields $l = k + \Omega m$. This defines the rotation rate Ω and the surface gravity is then defined by $\nabla (l^2) = \kappa\, l$. Finally, the potential α is the ordinary electrostatic potential at the horizon.

The new black hole solutions that have been discovered in the past few years have additional scalar or gauge fields that are not present in the case of electromagnetism. As the table below shows, we include as solutions cases that are unstable and even ones that have the 'wrong' spacetime signature. The reason for such a liberal approach is that black hole thermodynamics is a quantum phenomenon and the solutions may represent virtual processes.

Table 1. Black hole solutions with conventional gravity and a variety of other fields.

type	fields	matter Lagrangian	number	stable
dilaton	dilaton	$-\frac{1}{4}e^{-2\phi}F^2 - \frac{1}{2}(\nabla\phi)^2$	1	Yes
skyrmion	chiral	$-\frac{1}{4}k^2\mathrm{tr}A^2 - \frac{1}{32}g^2\mathrm{tr}(A \wedge A)^2$	N	Yes
sphaleron	gauge	$-\frac{1}{4}\mathrm{tr}F^2$	N	No
monopole	gauge/higgs	$-\frac{1}{4}\mathrm{tr}F^2 - (D\phi)^\dagger(D\phi) - V(\phi)$	N	Yes
vortex	gauge/higgs	$-\frac{1}{4}\mathrm{tr}F^2 - (D\phi)^\dagger(D\phi) - V(\phi)$	N	N/A

Dilaton (Gibbons and Maeda 1988)

The dilaton field appears in theories in which the spacetime dimension starts out larger than four. Special examples result from string theories. The charged black hole solutions with this matter action have non–constant scalar field values outside the event horizon. However the solutions are unique, provided that the combination $Qe^{-\phi}$ is interpreted as the charge. These solutions therefore only have hair in the weak sense of having non–trivial fields outside the horizon.

Skyrmion (Luckock 1987)

These solutions represent a black hole immersed inside a cloud of pions. In the Skyrmion model pions are represented by $SU(2)$ matrices, $U = \exp(iw(r)\sigma \cdot x)$. The Lagrangian in table 1 is given in terms of $A = g^{-1}U^{-1}dU$ with constants k and g. The topological solitons of this theory represent baryons, with the baryon number N given by the difference between w at infinity and at the origin, when there is no horizon. Black hole solutions have a horizon at $r = r_h$, and the value of $w(r_h)$ appears as two branches–an unstable and a stable branch– connected to each value of N (Luckock 1986, Droz 1991).

Sphaleron (Volkov and Gal'tsov 1989)

These are solutions with a Yang–Mills gauge field, typically with the gauge group $SU(2)$ (Volkov 1989, Bizon 1990, Künzle 1990). The fields look like

$$A = \frac{1 - w(r)}{2g}U^{-1}dU, \quad U = e^{i\sigma \cdot x}, \tag{2}$$

with $w(r) \to \pm 1$ as $r \to \infty$. The solutions are unstable (Straumann 1990) but lie midway between topologically distinct vacua and can represent tunnelling.

Monopole (Breitenlohner, Forgács and Maison 1992)

With a Higgs field as well as the gauge field there are a class of monopole solutions in which the Higgs field does not take the vacuum value outside of the event horizon (Breitenlohner 1992, Lee 1992). The gauge fields take the same form as for the sphaleron, but with $w(r) \to 0$ as $r \to \infty$, and the Higgs doublet field is of the hedgehog form $\phi = \chi(r)x \cdot \sigma/r$.

Vortex (Dowker, Gregory and Traschen 1992)

These solutions with a Yang–Mills gauge field and Higgs field are unlike the rest in that they have Riemannian spacetime signature $(++++)$. The reason for including these solutions is entirely due to the quantum nature of black–hole thermodynamics. For this reason these solutions are often described as examples of 'quantum hair' (Coleman 1991). The gauge and Higgs fields for a $U(1)$ gauge group are

$$A = w(r)dt, \quad \phi = \chi(r)e^{ig\gamma t}. \tag{3}$$

For a Higgs charge ng and fermion charge g, $w \to \gamma/n$ as $r \to \infty$. The solutions are topologically non–trivial only if $n > 0$, a rather 'non–standard' charge for the Higgs field. There is a winding number $0 \leq N < n$ for the Higgs phase factor, i.e $\gamma = 2\pi N/g\beta$.

3 Black Hole Thermodynamics

The remarkable extension of thermodynamics to black holes is based upon the laws of classical black hole mechanics (Bardeen 1973):

(0) The event horizon is described by a quantity κ, the surface gravity.

(1) Under external influences,

$$\delta M = \tfrac{1}{4}(\kappa/2\pi)\delta\mathcal{A} + \text{work terms.} \tag{4}$$

(2) The area increase law: $\delta\mathcal{A} \geq 1$.

(3) The limit $\kappa = 0$ cannot be reached in a finite time.

These laws are dependent upon a number of topological assumptions and energy conditions, e.g. the weak energy condition $T_{ab}l^a l^b \geq 0$ for null vectors l. Until recently the first law had only been investigated for particular models, and this is even more so for the third law (Israel 1992).

The main thermodynamic result missing from the classical laws is the statement of thermal equilibrium. This is supplied by Hawking's famous result,

(0) A black hole can be in thermal equilibrium with radiation at the Hawking temperature.

In order to establish this result it is necessary to look at quantum field theory on a black hole background. Underlying the result is the periodicity of black hole solutions in imaginary time which is analagous to the behaviour of thermally averaged Green functions. The period $\beta = 2\pi/\kappa$ is fixed by regularity of the metric at the event horizon.

The first law enables us to relate the entropy of a black hole to the area of the event horizon. It is important to establish whether the first law is still valid for the black hole solutions that are mentioned in table 1, and the recent work of Bob Wald goes some way towards this end (Wald 1993). It should also be born in mind that the equilibrium is usually unstable and only a subset of thermodynamics is strictly valid. Some discussion of this issue and attempts towards a better theory of black–hole thermodynamics can be found in the work of York (Brown 1991).

Another way to calculate the entropy is to employ the partition function, for example for the grand canonical ensemble. Calculation of the entropy $S(\beta, \mu)$ then proceeds as follows:

(1) Define the partition function

$$Z = \text{tr}(e^{-\beta M - \mu Q}) \tag{5}$$

by a path integral over field configurations with Riemannian geometry, period β and potential μ.

(2) Expand the partition function about a solution to the classical equations,

$$Z \sim (\det \Delta)^{-1/2} e^{-I(\beta, \mu)}. \tag{6}$$

The determinant contains the contribution from the radiation gas and the classical black–hole action I represents the coherent fields (Hawking 1979).

(3) Use a canonical decomposition of the action to show that for Einstein gravity plus matter (Moss 1992),

$$I = \beta M - \tfrac{1}{4}\mathcal{A} - \beta\Omega J + \beta\,\alpha_h Q_h - \beta\,\alpha_\infty Q_\infty \tag{7}$$

All of the quantities in this expression are measured at the horizon or at infinity. The gauge potential plus a topological term appears as α.

(4) Read off the thermodynamic potential $W = -T \ln Z$, and compare the result with

$$W = M - \beta^{-1}S - \sum \mu_i N_i \tag{8}$$

to deduce that the contribution to the action from the black hole is $S = \frac{1}{4}\mathcal{A}$. (If there is more than one black hole solution then the total entropy would include contributions from each one).

It is step (3) that contains most of the technical calculation. We want a formula for the action of a black hole that only depends on local terms at the horizon and infinity. We use the basic property of black hole solutions that they have a 'timelike' killing vector k, a spacelike killing vector m and an event horizon $\mathcal{S}_h$.

Begin by constructing spaces Σ of constant time which stretch from the event horizon to infinity and pick a tetrad. The coefficients of this tetrad in a coordinate basis define the lapse and shift functions N and N^i,

$$\omega^0 = N\,dt, \quad \omega^m = \omega_i{}^m(N^i dt + dx^i). \tag{9}$$

The surface metric $h_{ij} = \omega_i{}^m \omega_j{}^m$ summed over m.

Take as an example the matter Lagrangian density for scalar electrodynamics, with scalar field ϕ and gauge potential A_ν. The momentum conjugate to the spatial components of the gauge potential is the electric field density E^i. The Lagrangian has a canonical decomposition into

$$\mathcal{L}_m N\sqrt{h} = \pi\dot{\phi}^\dagger + \pi^\dagger\dot{\phi} + E^i\dot{A}_i - A_t(E^i{}_{,i} - \rho) - N\mathcal{H} - N^i\mathcal{H}_i + (E^i A_t)_{,i}. \tag{10}$$

The boundary terms is this expression are crucial.

The charge density $\rho = -i(\pi^\dagger\phi - \phi^\dagger\pi)$, and the magnetic field $B = h^{1/2}\nabla \wedge A$. The contributions to the Hamiltonian from each of the fields are tabulated below.

Table 2. Hamiltonian and momentum densities for various fields.

fields	$\mathcal{H}$	$\mathcal{H}_i$
scalar	$h^{-1/2}\pi\pi^\dagger + h^{1/2}\nabla\phi^\dagger \cdot \nabla\phi$	$\pi^\dagger\phi_{,i} + \pi\phi_{,i}{}^\dagger$
vector	$\frac{1}{2}h^{-1/2}(E \cdot E + B \cdot B)$	$h^{1/2}E \wedge B$
dilatonic	$\frac{1}{2}h^{-1/2}(E \cdot D + B \cdot H)$	$h^{1/2}E \wedge H$

Time translation symmetry *up to a gauge transformation* implies that $\dot{\phi} = ig\gamma\phi$ and $\dot{A}_i = \gamma_{,i}$. Set $\alpha = \gamma + A_t$. The matter action coming from integrating equation 10 becomes simply

$$I_m = \beta \int d\mu(\Sigma)(N\mathcal{H} + N^i\mathcal{H}_i) + \beta\,\alpha_h Q_h - \beta\,\alpha_\infty Q_\infty. \tag{11}$$

In the vortex solution, for example, the charge is a constant and $Q_h = Q_\infty$. At the horizon, $A_t = 0$ and therefore $\alpha_h = \gamma$, whilst at infinity $\alpha_\infty = 0$.

Canonical decomposition of the Einstein action can be used to derive a similar result for the gravitational part of the action. The details will be omitted because it is also possible obtain the result more easily from killing vector identities (Gibbons 1977,Moss 1992),

$$I_g = \beta M - \frac{1}{4}\mathcal{A} - \Omega J - \beta \int d\mu(\Sigma)(N\mathcal{H} + N^i\mathcal{H}_i). \tag{12}$$

The last term in this part of the action cancels with the identical term in the matter action to leave the result for the total action 7 that was quoted above.

The result extends to any matter field which has a canonical decomposition, and the dilaton theory is an example. If we set

$$D = e^{-2\phi}E \quad B = e^{2\phi}H, \tag{13}$$

then Maxwell field part of the Hamiltonian takes the form given in the table. The Maxwell equations take the natural form for electrodynamics of continuous media, $\nabla \cdot D = \rho$ etc., together with the dilaton field equation $\nabla^2\phi = \mathcal{H}\sqrt{h}$.

Figure 1: These Penrose diagrams show different possibilities where the singularity can be invisible or visible

Fundamental departures between the entropy and area relationship do occur when the gravitational part of the theory is modified. One example is to include terms in the Lagrangian that have two or more powers of the curvature tensor (Jacobson 1993, Visser 1993). In these cases the entropy is modified,

$$S = \tfrac{1}{4}\mathcal{A} + 4\pi \int d\mu(\mathcal{S}_h)\frac{\partial \mathcal{L}^{extra}}{\partial R_{\mu\nu\rho\sigma}}g^{\perp}_{\mu\rho}g^{\perp}_{\nu\sigma}, \tag{14}$$

where $g^{\perp}_{\mu\rho}$ is the projection of the metric orthogonal to the horizon.

4   Cosmic Censorship

Can we travel into a black hole and through into another universe, seeing the singularity, closed timelike curves etc. on the way? For ordinary black holes, surrounded by empty space, it seems that a singularity forms at the Cauchy horizon which prevents passage throught the hole (Israel 1992). This is caused by radiation from the exterior, which becomes infinitely blue–shifted near to the horizon resulting in a diverging energy density.

The situation with a cosmological constant is different, as shown in the figure. Light rays close to the Cauchy horizon have travelled from a cosmological horizon and not from infinity. The difference in surface gravities of the two horizons determines whether the wavelength of the light rays gets red or blueshifted, with redshift and Cauchy horizon stability when the cosmological horizon has the larger surface gravity (Mellor 1990, Brady 1992).

Both charged and rotating black hole solutions can be generalised to include a cosmological constant Λ (Carter 1973). The metric for a rotating hole is quite complicated but simplifies a little because of a symmetry between the radial coordinate r and colatitude coordinate $\mu = a\cos\theta$,

$$ds^2 = \rho^2(\Delta_r^{-1}dr^2 + \Delta_\mu^{-1}d\mu^2) + \rho^{-2}\Delta_\mu\,\omega^1 \otimes \omega^1 - \rho^{-2}\Delta_r\,\omega^2 \otimes \omega^2, \tag{15}$$

with one–forms,

$$\chi^2\omega^1 = dt - a^{-1}\sigma_r^2 d\phi, \quad \chi^2\omega^2 = dt - a^{-1}\sigma_\mu^2 d\phi. \tag{16}$$

The metric solution is parameterised as follows,

$$\sigma_r = (a^2 + r^2)^{1/2}, \quad \Delta_r = (a^2 + r^2)(1 - \tfrac{1}{3}\Lambda r^2) - 2Mr \tag{17}$$

$$\sigma_\mu = (a^2 - \mu^2)^{1/2}, \quad \Delta_\mu = (a^2 - \mu^2)(1 + \tfrac{1}{3}\Lambda\mu^2) \tag{18}$$

Figure 2: Penrose diagram of Kerr–de Sitter ($\theta = \pi/2$).

and

$$\rho^2 = r^2 + \mu^2, \quad \chi^2 = 1 + \tfrac{1}{3}\Lambda a^2. \tag{19}$$

The fully extended spacetime has many asymptotic regions and the extremal sections are infinite chains of three–spheres with black holes at their antipodes. In a situation where a rotating body collapsed to form a black hole, part of the Penrose diagram would represent the spacetime outside of the collapsing matter.

There are three basic types of horizon where the function Δ_r vanishes:

$\mathcal{S}_1$ cosmological horizon
$\mathcal{S}_2$ black hole event horizon
$\mathcal{S}_3$ black hole Cauchy horizon

Perturbation theory can be used to study stability of the spacetime (Chambers 1993). The analogue of the Starobinsky–Teukolsky equations for the radial modes $\mathcal{R}$ of a field with spin s, frequency ω and angular quantum number m are

$$\left(\mathcal{D}_{-s/2} \Delta_r \mathcal{D}_{s/2}^\dagger + 2(2s-1)i\omega\chi^2 \right) \mathcal{R} = (\nu + s)(\nu - s + 1)\mathcal{R}. \tag{20}$$

The gravitational case includes an additional cosmological term,

$$\left(\mathcal{D}_{-1} \Delta_r \mathcal{D}_1^\dagger + 6i\omega\chi^2 - 2\Lambda r^2 \right) \mathcal{R} = (\nu + 2)(\nu - 1)\mathcal{R}, \tag{21}$$

with radial derivatives

$$\mathcal{D}_n = \frac{\partial}{\partial r} + \frac{iK_r}{\Delta_r} + n\frac{\Delta_r'}{\Delta_r}, \quad K_r = \chi^2(am - \sigma_r^2\omega). \tag{22}$$

The equations for the angular modes, which determine the eigenvalues $(\nu + s)(\nu - s + 1)$, are obtained by replacing r by $i\mu$ in the radial equations. These equations reduce the stability

Figure 3: The parameter space of the rotating black hole.

analysis to a series of scattering problems, but the most important feature is the combined blue–shift of the ingoing radiation, $e^{\kappa_3 v}$ in the interior and $e^{-\kappa_1 v}$ outside, where v is a coordinate that becomes infinite at the Cauchy horizon. The Cauchy horizon is stable when the product is finite, i.e. $\kappa_3 < \kappa_1$.

The stability region is shown in the figure above, parameterised by the mass M and rotation parameter a scaled by the cosmological constant. The range of parameters for which there are three horizons lies inside the triangular region. On the outer borders two of the radii are equal, $r_2 = r_3$ on OA and $r_1 = r_2$ on AC, but the spacetimes can still be defined. Corresponding surface gravities also vanish there. Interior lines denote the cases where $\kappa_1 = \kappa_3$ (the lower line OA) and $\kappa_1 = \kappa_2$ (along OB). The narrow area between the two lines OA is the stability region.

Although the region is tiny for small values of the cosmological constant it would be possible to force the parameters of the hole in the right direction by increasing the rotation rate. If the third law of black hole mechanics is valid then there is an upper limit to the rotation rate at the upper line OA in the figure.

In fact there is an interesting violation of the third law of black hole mechanics for de Sitter black holes (Brill 1993). This is easiest to see for the charged case. The line corresponding to OB in the figure is given in the charged case by the condition $Q = M$ (Mellor 1989). The usual centrifugal barrier around $r = 0$ is absent for particles which have equal charge and mass. The singularity is no longer 'repulsive'. If a shell of this type of matter is dropped into a black hole, with parameters close to point B, the mass can be increased beyond B (still with $Q = M$) and the horizon stripped from the hole. This is a violation of the third law because $\kappa_2 = 0$ at B.

The modification of a charged black hole that would lead to stability involves an increase in the charge to mass ratio and therefore the validity of the third law for this case is an open issue. It remains to be seen whether Cauchy horizon stability is a 'practical' proposition in a universe which has a very small cosmological constant.

Finally, there is the possibility that the cosmological constant was once very large and responsible for a period of inflationary expansion. The issues of interest in this situation are the stability of the whole universe rather than just the Cauchy horizon and the existance of

'no hair' theorems that generalise the results for black holes in empty space (Hawking 1982).

The generalisation of Price's theorem to de Sitter space takes the form:

The only stationary solutions to the massless wave equations with spin $s = 0, 1, 2$ on a spherically symmetrical black hole background in de Sitter space have angular momentum less than s.

In fact the modified Starobinsky–Teukolsky equations given above have no stationary solutions at all when the rotation parameter $a = 0$. Take the equation,

$$\left(\mathcal{D}_{-s/2} \Delta_r \mathcal{D}^\dagger_{s/2} - (\nu + s)(\nu - s + 1) \right) \mathcal{R} = 0. \tag{23}$$

Multiply by the complex conjugate mode solution and integrate between the horizons r_1 and r_2,

$$\int_{r_2}^{r_1} dr \mathcal{R}^\dagger \left(\mathcal{D}_{-s/2} \Delta_r \mathcal{D}^\dagger_{s/2} - (\nu + s)(\nu - s + 1) \right) \mathcal{R} = 0. \tag{24}$$

For physically reasonable solutions it would be necessary for both $\mathcal{R}$ and $\mathcal{D}_0 \mathcal{R}$ to be regular at the horizons. For the case $a = 0$, integration by parts gives

$$\int_{r_2}^{r_1} dr \left(-\Delta_r^{(1-s)} (\Delta_r^{s/2} \mathcal{R})^{\dagger\prime} (\Delta_r^{s/2} \mathcal{R})' - (\nu + s)(\nu - s + 1) \mathcal{R}^\dagger \mathcal{R} \right) = 0. \tag{25}$$

The function Δ_r is positive in this range of integration and the value of ν is a non–negative integer when $a = 0$.

The only consistent solutions to the Starobinsky–Teukolsky equations with $\nu \geq s$ are therefore $\mathcal{R} = 0$. Some field components are not fully determined by solutions to the Starobinsky–Teukolsky equations and have to be considered seperately (Chambers 1993). They are only determined up to a solution of

$$\mathcal{D}_0(r^{1+s}\phi) = 0 \tag{26}$$

and they have angular momentum less than s. (For those familiar with Newman–Penrose formalism, these fields are the Maxwell scalar ϕ_1 or the perturbation of the Weyl tensor component Ψ_2.) When $s = 1$, the complex scalar is composed of the radial electric and magnetic fields and the two real constants of integration are the electric and magnetic charges of the black hole. When $s = 2$, ϕ is the perturbation to the Weyl tensor and the constants of integration are the mass perturbation and the angular momentum.

The black hole solutions show very explicitly that a period of inflation need not lead to a universe that is fully homogeneous. On the other hand, any observer who does not fall into the singularity sees a local spacetime that comes to look exactly like de Sitter space and becomes increasingly isotropic. Furthermore, most of the spacetime behaves this way and it seems that inflation will still work even if the universe began with large inhomogeneities (Shiromizu 1993).

The decay of perturbations can be seen explicilty in the case of de Sitter space $a = M = 0$ (Mellor 1990). The modified Starobinsky–Teukolsky equations can be written as a scattering problem using the coordinate r^*, where $r = \alpha \tanh(r^*/\alpha)$ and $\alpha^2 = 3/\Lambda$. Then the perturbation equations are simply

$$\left(\frac{d^2}{dr^{*2}} + V(r) \right) r\mathcal{R} = 0 \tag{27}$$

where

$$V(r) = \left(\omega + \frac{is}{r} \right)^2 - (\nu(\nu + s + 1) + s(s - 1)) \frac{\Delta_r}{r^4}. \tag{28}$$

The solutions can be written in terms of Jacobi polynomials,

$$r\mathcal{R} = \left(1 + \frac{\alpha}{r} \right)^{(i\alpha\omega - s)/2} \left(1 - \frac{\alpha}{r} \right)^{(i\alpha\omega + s)/2} P_\nu^{(i\alpha\omega + s)/2, (i\alpha\omega - s)/2}(\alpha/r) \tag{29}$$

This picture shows in particular how any given perturbation leaves the future horizon of any observer due to the representation as a wave in the r^* and t coordinate system. Therefore a set of perturbations that where sufficiently well controlled in ω and angular momentum would always die away from the local region of an observer.

5   Conclusions

It is quite clear that non–abelian gauge theories lead to violations of the no–hair theorems, but in all of the examples produced so far these violations are not very severe. Most importantly of all, the basic tenets of black hole thermodynamics still hold good and the simplicity of the structure of the event horizon is maintained. This shows up clearly in the formula for the action, which depends only on quantities measured either at the event horizon or at infinity, and suggests that the solutions are unique given the values of the angular momentum J, the mass M and *both* of the charges Q_h and Q_∞.

Cosmological 'no hair' theorems are important in the inflationary universe scenario. The stability analysis of black holes solutions gives us a way to approach the evolution of the early universe and to address questions that involve large inhomogeneities, which may have been present at the origin of our universe.

The conclusions reached from the study of black holes in de Sitter space present far more of a challange to general relativity. The existence of naked singularities in a spacetime with reasonable matter, if we accept the cosmological constant, gives us a good reason for studying the problems of quantum gravity.

6   List of Symbols

$A \cdot B = h^{ij} A_i B_j$ $\qquad$ h_{ij}=spatial metric
$A \wedge B = \epsilon^{ijk} A_j B_k$ $\qquad$ $d\mu(\mathcal{M})$ =volume measure on $\mathcal{M}$
$\hbar$=Planck constant $\qquad$ S=entropy
c=velocity of light $\qquad$ β=inverse temperature
G=Newton constant $\qquad$ $\mathcal{A}$=horizon area

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