SHETRAN Water Flow Component, Equations and Algorithms

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1 General Introduction

1.1 Introduction to SHETRAN

SHETRAN is a physically-based, distributed, deterministic, integrated surface and subsurface modelling system, designed to simulate water flow, sediment transport, and contaminant transport at the catchment scale (Ewen et al., 1995)

1.2 Catchment flow modelling

SHETRAN is designed primarily to model catchments, normally consisting of a catchment basin with a zero-flow boundary (the watershed, or catchment divide), and a channel network feeding surface and subsurface responses to precipitation to a single outflow reach of channel, although a full range of boundary conditions can be invoked to allow representation of, for example, small experimental plots.

Meteorological inputs to the catchment are: precipitation falling as rain or snow; measured or calculated potential evapotranspiration; and heat budgets used in calculating rates of snowmelt. Net precipitation to the ground surface is calculated from incoming precipitation and snowmelt, with allowance for interception, evaporation, and drainage from a vegetation canopy. Actual rates of evapotranspiration are calculated as a function of dynamic soil moisture conditions. Evaporation rates from leaf surfaces of the vegetation, from the soil surface, and from free water surfaces are calculated.

Infiltration into the ground surface occurs from either net precipitation or from surface water. Distribution of soil moisture content and tension in the unsaturated zone, and recharge to the saturated zone, are calculated. Saturated zone flows are calculated for a heterogeneous, anisotropic, unconfined aquifer. Exchange flows between the aquifer and a partly penetrating channel are computed, with or without a low permeability channel lining.

Surface water is generated by either infiltration excess or saturation excess mechanisms, and is routed into the channel as sheet overland flow. Channel routing takes place through the channel network, through variable cross-section channel reaches, connected in any topology (distributaries as well as tributaries can be included, allowing looped networks). Backwater effects and overbank flooding can be modelled, at a level relevant to catchment modelling; i.e. SHETRAN is not a detailed hydraulic model of localized channel flow behaviour. Additional sources and sinks to the channel are from aquifer exchanges through the channel bed and banks, direct infiltration and evaporation at a dry channel bed, and direct extraction of channel water by plants.

1.3 Structure of SHETRAN

SHETRAN has a modular design, in which each module or component is used to represent the different physical processes of the hydrological cycle in each part of the catchment, or of sediment transport or contaminant transport. SHETRAN V3.4 has five components:

FR frame;
WAT water flow;
SY sediment transport;
CM contaminant transport;
SHE library.

The frame component consists of two modules: the frame module (FR), and the bank (BK).
The water flow component consists of 4 modules:

ET evapotranspiration/interception;
OC overland/channel;
VSS variably saturated subsurface
SM snowmelt.

2 Frame Module

2.1 Introduction

The Frame module of SHETRAN provides the overall control of a simulation including its initialisation, simulation timesteps, results output, and termination. The following functions are performed by the module:

(i) general data are read from the Frame input file;
(ii) element numbers, sizes and areas are assigned to each discretization element (grid, bank or channel link), linkages between elements are set up, and the total catchment area is calculated and checked;
(iii) reading and initialisation of input data for all other modules is controlled;
(iv) the computational timestep is calculated, based on the current meteorological data and the timestep reduction criteria;
(v) each of the flow modules, and sediment and contaminant components, is called for each timestep, where required;
(vi) internal boundary data exchanges between the modules are calculated;
(vii) summary results are printed to the ASCII output file;
(viii) detailed simulation output is written to the HDF files,for display.
(ix) the simulation is terminated and final statistics reported.

2.2 Computation Elements

There are three types of spatial discretization elements set up for a SHETRAN simulation (see Fig. 2.1):

(i) Grid elements. These are the basic computational element making up most of the catchment area; they are rectangular in shape, based on a mesh-centred discretisation in both x and y directions (i.e. the boundary between two grid elements is at the mid-point between the two computational nodes);
(ii) Channel links. The catchment channel network is described by channel links running along the edges of grid elements; each link has the same length as the grid that it runs adjacent to, and can be assigned an individual width and crosssection; the computational node is at the centre of the channel link; it is assumed that channel widths are small relative to the adjacent grid elements;
(iii) Bank elements. Bank elements are narrow strips of land along each side of channel links, with length equal to the length of the link, and a fixed width; the computational node is at the centre of the bank element; it is assumed that bank elements are narrow relative to the adjacent grid elements.

2.3 Component and Module Execution

For every simulation, the three basic flow modules are automatically included (variably saturated zone (VSS), evapotranspiration/interception (ET), overland/channel (OC)). For each timestep, the VSS and OC modules are called in sequence (see Fig. 2.2). Owing to the close interaction between the VSS, ET, and optionally the SM modules, these three are coupled at each timestep.. The SY and CM components are called, if required, at the end of a timestep, when all flow variables at time level n+ 1 are available.

2.4 Timestep Calculations

Each of the components of SHETRAN uses the same timestep, but the timestep can vary throughout a simulation. During dry periods the timestep can be relatively large (a basic timestep of one or two hours is often used). During a storm event rates of infiltration into the ground and surface runoff can both vary rapidly, so a smaller timestep must be used. The timestep is modified in SHETRAN based on user-defined criteria for rates of rainfall. A maximum amount of rainfall allowed in any one timestep is specified. If the rainfall rate within the current timestep exceeds this figure, the timestep is reduced (see example in Fig. 2.3). At the end of the rainfall event, the timestep is increased by a fixed ratio until the basic timestep is again reached.

2.5 Data Passed Between Flow Modules

During a SHETRAN simulation, each of the flow modules uses head and flow boundary data from the other modules. These internal boundary conditions are passed between the modules within a timestep during the normal sequence of execution, and from one timestep to the next (see Fig. 2.4). Interactions between modules takes place, as far as possible, using data at time level n+1 (i.e. at the end of a timestep). The flow component passes data to the sediment and contaminant components, but does not receive data in return.

2.6 Results Output

The normal method for extracting and presenting results is by writing to HDF files. These are read and displayed by free HDF Viewer software. **** .The available methods of display include time-series of values at any point within the three-dimensional catchment grid, spatial distributions at any time as areal grid plots, slices through the subsurface at any depth, and vertical profiles of subsurface variables. A wide range of flow, sediment, and contaminant variables are available.

Initialisation data, diagnostic information, discharge at the outlet and summary statistics, are written to formatted ASCII files.

3 Evapotranspiration / Interception Module

3.1 Introduction

The evapotranspiration/interception (ET) module calculates:

- potential evapotranspiration from vegetation, soil, and free water surfaces;
- evaporation from a wetted canopy;
- evaporation from bare soil and from dry channels;
- evaporation from a free water surface;
- plant transpiration;
- plant uptake from soil in the root zone;
- canopy storage;
- drainage from a canopy;
- net precipitation beneath a canopy.

It is assumed that the temperature is above 0°C and that there is no snowpack. Otherwise, responsibility for modelling the processes is devolved to the snowmelt (SM) module.

Depending on the availability of data several different versions of the model can be applied, involving finer or coarser representations of the processes and requiring more or fewer data.

3.2 Evapotranspiration

The evapotranspiration model calculates actual evapotranspiration and translates it into a loss term describing uptake of water by plant roots and its transpiration for dry partially wet, and fully wet conditions. The loss term is then used in the calculation of soil moisture changes by the variably saturated subsurface module.

Both actual and potential evapotranspirations are calculated. Potential evapotranspiration is that which occurs when the supply of water to the plant/sail system with a dry canopy is unlimited. It is assumed to depend only on meteorological conditions and the radiational and aerodynamic properties of the soil and plant cover, and to be independent of flow conditions within the soil and of the physiological restrictions on evapotranspiration within the root/plant system. It is calculated using Penman's combined energy balance/turbulent transfer equation (Penman, 1948) as:

$$E_p = \frac{R_n \Delta + \frac{\rho c_p \delta e}{r_a}}{\lambda(\Delta + \gamma)} \qquad 3.1$$

where:

E_p = potential evapotranspiration

R_n = net radiation

Δ = rate of increase with temperature of the saturation vapour pressure of water at air temperature

ρ = density of air

c_p = specific heat of air at constant pressure

δe = vapour pressure deficit of the air

r_a = aerodynamic resistance to transport of water vapour from the canopy to a plane 2 m above it

λ = latent heat of vaporization of water

γ = psychrometric constant given by

$$\gamma = \frac{pc_p}{\sigma \lambda} \qquad 3.2$$

where:

p = atmospheric pressure

σ = ratio of density of water vapour to density of air (approximately 0.622)

The value of the aerodynamic resistance r_a can be given as a constant, or else calculated from the wind velocity profile if the necessary data are available. In the latter case, r_a is defined by

$$r_a = \frac{\rho u}{\tau_o} \qquad 3.3$$

where τ_0= the shear stress between the air flow and the boundary and u = the windspeed 2 m above the canopy. If the standard logarithmic boundary layer equation is used to represent the velocity profile of the wind (ASCE, 1976) then

$$r_a = \frac{1}{\kappa^2 u} [\ln(\frac{z - d}{z_o})]^2 \qquad 3.4$$

where d = zero plane displacement (assumed to be 0.75 h); z_0 = roughness height (assumed to be 0.1h); h = the vegetation stand height; z = the height of the anemometer (z = h + 2m for the windspeed measured 2m above the canopy); κ = von Kármán constant (=0.41).

The zero plane displacement is a mathematical correction introduced to ensure that the relationship between velocity and the logarithm of (corrected) height is linear. It is required because the velocity may not be zero at the datum level $z = z_0$, owing to the effects of boundary roughness and flow turbulence. Thus velocity is assumed to be zero at the level $z = d + z_0$.

Actual evapotranspiration should have the potential rate as an upper limit and otherwise be reduced by restrictions in the supply of water from the soil to the roots or by physiological controls (such as stomatal resistance) within the plants. However, even with an unlimited water supply the factors affecting evapotranspiration are not well understood and an operational model of the process must inevitably contain simplifications. Two approaches are used in SHETRAN, giving flexibility and allowing the model to be adapted to whatever is known of local conditions. In the first (PenmanMonteith model, section 3.2.1), actual evapotranspiration at subpotential rates is assumed to be limited by vegetational factors, particularly the stomatal resistance to movement of water. In the second (section 3.2.2), the limitation is assumed to be due only to the resistance of the unsaturated soil to water movement.

3.2.1 Model based on vegetative control of actual evapotranspiration (Penman-Monteith model)

Monteith (1965) modified Penman's combined energy balance/turbulent transfer equation (Eq. 3.1) for predicting potential evapotranspiration by including a factor accounting for the resistance to water movement of the evaporating surface. The resistance can be thought of as a physiological resistance, closely related to the average stomatal resistance to vapour flux, and is described by a canopy resistance factor. The equation used in the model is (Monteith, 1965):

$$E_a = \frac{R_n\Delta + \dfrac{\rho c_p \delta e}{r_a}}{\lambda\left[\Delta + \gamma\left(1 + \dfrac{r_c}{r_a}\right)\right]} \qquad 3.5$$

where

E_a = actual evapotranspiration

r_c = canopy resistance to water transport from some region within.or below the transpiring surface to the surface itself, being zero for a wet canopy (evaporation is already occurring at the potential rate) and equal to average stomatal resistance in dry conditions

The other terms are as defined for Eq. 3.1. In the model, r_c can be defined as a constant or as a function of soil moisture tension.

3.2.2 Model based on soil moisture control of actual evapotranspiration

For this alternative to the Penman-Monteith equation it is assumed that the conditions of unsaturated flow through the soil limit the losses from the soil to the roots and from the ground surface. A simplified approach to the calculation of evapotranspiration is presented by Feddes et al. (1976) in which the complexities of the soil/plant/atmosphere system are represented by the following lumped analogue.

Under conditions drier than some wilting point ψ_w and wetter than some anaerobis point ψ_s (where ψ = soil moisture tension), plants do not take up water from the soil so actual evapotranspiration is zero. Between ψ_s, and some arbitrary pressure head ψ_L at which soil water begins to limit plant growth, water uptake is considered to take place at the potential rate, i.e. actual evapotranspiration equals potential evapotranspiration calculated by Eq. 3.1. For $\psi_w < \psi < \psi_L$ it is assumed that actual evapotranspiration varies linearly as a proportion of the potential evapotranspiration according to soil moisture tension ψ (Fig. 3.2).

The relationship between the ratio of actual to potential evapotranspiration and soil tension is specified in SHETRAN as a table of Ea/Ep against ψ, allowing a more flexible functional relationship than that shown in Figure 3.2 to be used if required.

3.2.3 Calculating soil loss rate

Soil evaporation and plant transpiration are calculated as sink terms for the three dimensional subsurface flow equation (Eq. 5.2). these terms depend on the proportion of ground covered by vegetation, the total leaf area of the vegetation, the vertical root density distribution function and the amount of water storage on the canopy. Several of these parameters vary seasonally.

It is assumed that transpiration occurs at the actual rate and from the total leaf area. However, if the canopy is totally wetted (i.e. the canopy storage C exceeds the canopy storage capacity S, which is the amount of moisture needed to wet the canopy totally) evaporation of intercepted water occurs at the potential rate so transpiration must be zero. Evaporation from bare soil is assumed to occur at the actual rate weighted according to the proportion of ground which is not covered by vegetation.

3.3 Interception of Rainfall

The interception model calculates net rainfall reaching the ground through the canopy, amount of water stored on the canopy, and evaporation from the canopy. Interception of liquid precipitation only (i.e. no snow or fog drip) is considered. Interception of snow is calculated in the snowmelt model.

The model is based on that developed by Rutter et al. (1971/72, 1975) and is essentially an accounting procedure for the amount of water stored on the canopy, C. Although developed initially for trees it is used here for all vegetations (trees and grass) since the physical principles are the same. The canopy is considered to have a surface storage of capacity S which is filled by rainfall and emptied by evaporation and drainage. S may be interpreted as the minimum depth of water required to wet all canopy surfaces. Intercepted rainfall depends on the proportion of the ground which, in planview, is hidden by vegetation, i.e. hidden by branches and leaves. This varies seasonally but has a maximum value p where (1-p) is the proportion of ground consisting of bare soil or rock which is never covered by vegetation. Evaporation of intercepted water on the other hand is assumed to occur from the total leaf area (including leaves which lie below other leaves) and therefore depends on the ratio p' of total leaf area to area of ground covered by vegetation. (Thus pp' is the ratio of leaf area to total ground area).

However, since the maximum volume rate of evaporation which can be sustained by the air is equal to the depth rate of evaporation multiplied by the plan area of ground covered by vegetation, the dependency on p' is valid only while the ratio is less than unity. Thus, in effect, both rainfall interception and evaporation of intercepted rainfall, vary in the same way since for p' < 1 they both vary with pp', while for p' > 1 they are both constrained by the limiting value of p. The division of ground between that covered by vegetation and that consisting of bare soil is illustrated in Fig. 3.3.

When the depth of water C on the canopy equals or exceeds S, the evaporation from the canopy is assumed to occur at the potential rate, E_p When C is less than S the rate is assumed to be E_p,.C/S. The rate of change of storage is then given by

$$\frac{\partial C}{\partial t} = Q - ke^{b(C-S)} \qquad 3.6$$

where
$Q = pp'P - pp'E_p\, C/S$ when $C \leq S$
$Q = pp'P - pp'E_p$ when $C > S$

$pp' = pp'$ when $p' \leq 1$
$pp' = p$ when $p' > 1$

and
C = depth of water on the canopy
S = canopy storage capacity
P = rainfall rate
p = proportion of ground in plan view hidden by vegetation at its maximum extent
p' = ratio of total leaf area to area of ground covered by vegetation
E_p = potential evaporation rate
k and b = drainage parameters
t =time

3.4 Numerical algorithms

The algorithms for the ET module are straightforward; the equations in this section are evaluated in the appropriate sequence, for each element in the catchment (there is no interaction between elements). The sequence of calculations is as follows.

1. Call the snowmelt module, if required. If the temperature is below zero, no further ET calculations are performed. If the temperature is above zero, and a snowpack exists, only canopy calculations are performed.

2. Calculate potential evapotranspiration

3. Calculate evaporation from intercepted storage, canopy storage, and drainage from the canopy

4. Calculate net precipitation as precipitation falling directly onto bare soil plus drainage from the canopy, weighted according to the proportion of ground covered by vegetation.

5. Calculate soil moisture loss from each cell in the root zone, based on one of the optional methods for calculating actual evapotranspiration. Sum the losses from all cells to give total transpiration. Fig 3.4 gives a diagrammatic representation of the transpiration model.

6. Calculate evaporation from the soil surface.

4 Overland / Channel module

4.1 Introduction

The overland/channel (OC) module calculates:

- the depth of surface water on the ground surface and in stream channel networks;
- the flow of surface water across the ground surface, along stream channel networks, and into or out of stream channels (including overbank flooding).

Both the overland and channel phases of the OC module are based on the diffusive wave approximation of the full St. Venant equations, allowing backwater effects to be modelled (the water surface slope is not neglected). The basic types of flows between elements are represented in Fig. 4.1. They are flows between any two overland flow elements (grid-grid, grid-bank, or bank-bank along the direction of the channel), between a channel link and the adjacent element (grid or bank), between two channel links (including flow at channel tributaries), and at catchment boundaries (grids or channel links). Figure 4.2 shows the processes modelling in the OC component

4.2 Basic Equations

The continuity equation for a general element (grid square, bank element, or channel link) can be written

$$\frac{\partial h_o}{\partial t} = \frac{1}{A}\left[\sum_{i=1}^{4} Q_i + Q_R\right] \qquad 4.1$$

where h,, is the water depth above ground in the element, A is the surface area of the element, Q; (i=1,4) are the lateral influxes (assumed to be positive into the element), and Q_R is the net vertical input to the element (net precipitation plus saturated flows to the surface less infiltration and evaporation). Note that, if more than two channel links join at a point (where two river tributaries join, or where a river channel splits into two), the flow into or out of the end of a channel link will be made up of a sum of two or more flows.

For each direction of overland flow (in the cartesian grid system used in SHETRAN), and for a linear channel reach, we have an equation derived from conservation of momentum, which can be written

(overland flow - x direction)

$$S_{fx} + \frac{\partial(z_g + h_o)}{\partial x} = 0 \qquad 4.2$$

(overland flow – y direction)

$$S_{fy} + \frac{\partial(z_g + h_o)}{\partial y} = 0 \qquad 4.3$$

(channel flow)

$$S_{fl} + \frac{\partial(z_g + h_o)}{\partial l} = 0 \qquad 4.4$$

where x, y are cartesian coordinates, l is the distance along the channel (in the x or y direction), z_g is ground or channel bed level (metres above datum), and S_{fx}, S_{fy}, S_{fl} are the friction slopes (Henderson, 1966) in the x, y and 1 directions respectively.

Assuming a Manning-type law (Henderson, 1966) for the friction slopes we have

$$S_{fx} = \frac{u_x^2}{K_x^2 h_0^{4/3}} \qquad 4.5$$

$$S_{fy} = \frac{u_y^2}{K_y^2 h_0^{4/3}} \qquad 4.6$$

$$S_{fl} = \frac{u_l^2}{K_l^2 h_0^{4/3}} \qquad 4.7$$

where u_x, u_y and u_l are the flow velocities in the x, y and l directions, and K_x, K_Y and K_l are the respective Strickler coefficients (the Strickler coefficient being the inverse of the Manning coefficient).

Combining Eqs. 4.2-4.4 and 4.5-4.7, the flow across an element boundary (between two elements of the same type, or between a grid square and a bank element) can be written

$$Q_x = u_x h_0 \, w_x = K_x \left[-\frac{\partial(z_g + h_o)}{\partial x} \right]^{1/2} w_x h_0^{5/3}$$
$$= \frac{K_x w_x h_0^{5/3}}{L_x^{1/2}} \left[z_u - z_d \right]^{1/2} \qquad 4.8$$

$$Q_y = u_y h_0 w_y = K_y \left[- \frac{\partial(z_g + h_o)}{\partial y} \right]^{1/2} w_y h_o^{5/3}$$
$$= \frac{K_y w_y h_0^{5/3}}{L_y^{1/2}} \left[z_u - z_d \right]^{1/2} \qquad 4.9$$

$$Q_l = u_l A_l = K_l A_l \left[- \frac{\partial(z_g + h_o)}{\partial l} \right]^{1/2} h_o^{2/3}$$
$$= \frac{K_l A_l h_0^{2/3}}{L_l^{1/2}} \left[z_u - z_d \right]^{1/2} \qquad 4.10$$

where w_x and w_y are the widths of the elements across the x and y directions respectively, L_x L_y and L_l are the distances between the centres of the two elements, A_l is the crosssectional area of the channel (a function of h_o), and z_u and z_d are the water surface elevations in the upstream and downstream elements respectively. Q_x and Q_y are represented by Q_{GRD} in Fig. 4.1, and Q_l is represented by Q_{LNK}.

The terms relating to fixed channel characteristics in Eq. 4.10 are written as a single variable, known as the conveyance of a channel section, a function of depth:

$$C = K_l A_l h_o^{2/3} \qquad 4.11$$

Since the value of conveyance is required for each reach of channel between two computational points (centres of channel links), whereas the values of K_l and A_l are given separately for each channel link, a weighted mean conveyance is used:

$$\bar{C} = \alpha C_u + (1 - \alpha)\, C_d \qquad (4.12)$$

where the C, and C_d are the values of conveyance at the upstream and downstream channel links and a is a weighting coefficient between 0 and 1. Similarly, weighted means for the values of $K_x h_0^{5/3}$ and $K_y h_0^{5/3}$ between two adjacent grid squares or bank elements are used in Eqs. 4.8 and 4.9.

4.3 Overbank Flow

Equations 4.8-4.10 describe flows between similar elements (ie. overland flow elements or channel elements). In addition, flows may occur between overland flow elements and channel elements, or vice-versa (overbank flooding). This can take one of a number of forms: inflow to the channel from adjacent grid elements or bank elements when the water depth in the channel is low and the adjacent element ground level is higher than the bank-full elevation of the channel link (Fig. 4.3 (a), (d) and (e)); zero flow when the adjacent element ground level is lower than the bank-full elevation of the channel link, and the channel does not overflow (Fig. 4.3 (b)); and overbank flow, when the water surface elevation in the channel is greater than in the adjacent elements (Fig. 4.3 (c)). Control of the flow between a grid element and the channel (allowing ponded water on the grid) can be imposed by setting either the channel bank-full elevation or the bank element ground surface elevation higher than the grid ground surface elevation.

The flow of water between a bank element (or a grid element if bank elements are not included in a simulation) and a channel link is described using a resistance equation for inflow into the channel. For overbank flow a resistance equation is also used if the adjacent ground level is higher than the channel bank elevation, while the standard flow equation for a broad-crested weir is used if the adjacent ground level is less than the channel bank elevation. This formulation allows the possibilities of non-drowned flow (when the channel water level is low compared with the bank elevation), and of drowned f low (when the channel water level is comparable with the bank elevation).

For the overbank flooding case, the flow is considered to be drowned if the downstream stage is greater than the critical depth (Henderson, 1966); ie. if

$$(z_d - z_b) > \frac{2}{3}(z_u - z_b) \qquad 4.13$$

where z_b is the bank elevation and z_u and z_d refer to the upstream and downstream water surface elevations (i.e. in the channel and the adjacent element). The bank is considered flooded, therefore, if

$$z_d > \frac{2z_u + z_b}{3} \qquad 4.14$$

For the drowned case, the flow between the bank element (or the grid element if no bank elements are present) and the channel link, is given by a broad-crested weir equation (Henderson, 1966) as

$$Q = \sqrt{2g}\, w(z_u - z_d)^{1/2}(z_d - z_b) \qquad 4.15$$

where w is the width of the ‘weir’ (i.e the length of the channel link), and g is the acceleration due to gravity.

For the undrowned case, the flow is given by (Henderson, 1966)

$$Q = 0.386\sqrt{2g}\, w(z_u - z_b)^{3/2} \qquad 4.16$$

In each of the above cases, the bank elevation z_b is defined by the channel-full elevation.

4.4 Boundary Conditions

Boundary conditions for overland flow can be specified as either prescribed head or prescribed flow. Head boundary data are given as a function of time, and can be defined for any grid or bank element, internal or on the catchment boundary. Flow boundary data are given as a function of time for grid elements on the catchment boundary only. The flow boundary data are given for the element adjacent to the boundary, and are assumed to be through one arbitrary boundary face of the element. Any boundary grid elements with no prescribed lateral fluxes, and all boundary bank elements, have a default zero flux boundary condition.

Boundary conditions for the channel system can either be given as a prescribed flux, or described by a weir equation (including a river and weir in parallel). All boundary data for the channel are given at the ends of a channel link. The upstream ends of a channel system wholly within a catchment are not hydraulically connected to any other element (grid, bank or link), and thus default to being a no-flow boundary. The downstream (outflow) boundary condition can be described as a prescribed flux, but normally a weir equation is used. Many different weir equations have been developed for different types of weir geometry (see Herschy, 1.978). The implementation in SHETRAN uses equations representing a flat-crested type of weir. Flow across the weir is considered to be drowned if the downstream stage is greater than the critical depth; ie if

$$(z_d - z_s) > s\,(z_u - z_s) \qquad 4.17$$

where z_d, z_s and z_u are the downstream, weir sill, and upstream water surface elevations respectively, and s is a submergence coefficient between 0 and 1. The flow over a drowned weir is then

$$Q = K\,(z_u - z_d)^{1/2}\,(z_d - z_s) \qquad 4.18$$

For an undrowned weir, where $(z_d - z_s) \leq s(z_u - z_s)$, the flow is given by

$$Q = K\,(z_u - z_s)^{3/2} \qquad 4.19$$

4.5 Numerical algorithms

An implicit finite difference formulation is obtained by expanding the first term on the right hand side of Eq. 4.1 as a Taylor series (to first order) at time level n+1. Substituting the expressions in Eqs. 4.8 to 4.10 gives a linearised set of equations in Δh, where Δh is the increment in water depths in each element over the computational timestep.

For the OC module (only), the elements are ordered by scanning a catchment grid from left to right along each row, including link and bank elements. Each row is represented by a submatrix in the coefficient matrix of the system of linear equations. The coefficient matrix then comprises a block tri-diagonal system (the system is block tri-diagonal since each row is adjacent to only the rows immediately above and below). The block tri-diagonal system is solved using a standard Thomas algorithm (Press et al.,1992).

The Δh's are then added to the water depths at time level n to give the final water depths at time level n+ 1.

5 Variably Saturated Subsurface Module

5.1 Model functional specification

The processes modelling in the variably saturated subsurface module can be seen in figure 5.1. A descriptive functional specification for the model is given below.

i) The model simulates fully three-dimensional flow in saturated and unsaturated single porosity / single permeability porous media. (Flow through fractured media can be represented for conditions where an EPM model is appropriate.)

ii) Flow through multiple layers of porous media of differing characteristics can be simulated; the layers can be laterally extensive, discontinuous, or of limited lateral extent (for example, clay lenses).

iii) Confined, semi-confined, and unconfined aquifers can be represented.

iv) Groundwater perching above low permeability lenses can be represented.

v) The lower boundary conditions are either a) a prescribed (time-varying) flux into the model, or b) a free drainage flux out of the model. The default is a no-flow boundary condition.

vi) The edge boundary conditions are either a) a prescribed (time-varying) head, or b) a prescribed (time-varying) flux, for each layer. The default is a no-flow boundary condition.

vii) The upper boundary conditions are either a) a prescribed (time-varying) head, when ponded surface water is present, or b) a mixed type boundary condition, when no surface water is present, and there is precipitation and evaporation at the ground surface. Evaporative fluxes are calculated in the SHETRAN evapotranspiration/interception module, and are a function of pressure potential. The model automatically switches between the two types of boundary condition.

viii) The boundary conditions at a stream channel bed are identical to those at the ground surface (either a prescribed head, when stream water is present, or a flux representing precipitation and evaporation, when the channel is dry).

ix) Seepage faces can be generated both at the intersection of layers with a stream channel, and at the ground surface.

x) A distributed sink term in the upper soil layers is included for plant transpiration. The (time-varying) flux is calculated in the SHETRAN evapotranspiration/ interception component, and is a function of pressure potential.

xi) A distributed sink term is included for well abstraction, as a prescribed (time-varying) flux. Simple control mechanisms are included to prevent unphysical abstraction from dry wells.

xii) Springs are represented as point discharges of groundwater directly into a stream channel.

5.2 Variables and Parameters

The dependent variable for the model is:

$\psi(\underline{x},t)$	pressure potential [L].

The independent variables are:

$\underline{x} = (x,y,z)$	space co-ordinates [L], where z is increasing positive upwards;
t	time [T].

Note that hydraulic head, $h(\underline{x},t)$ [L], is used for defining some of the boundary conditions. Hydraulic head and pressure potential are related by

$$h = \psi + z \quad . \tag{5.1}$$

A list of the parameters which must be defined by the user is given in Table 5.1. A list of the model variables used for initial and boundary conditions is given in Table 5.2.

parameter		varies with
d_B	thickness of stream bed sediments [L]	x,y[1]
C_s	spring discharge coefficient [L^2 T^{-1}]	-
$\underline{K}=(K_x,K_y,K_z)$	saturated hydraulic conductivity [LT^{-1}]	$\underline{x}$
K_B	saturated hydraulic conductivity of stream bed sediments [LT^{-1}]	x,y[1]
k_r	relative hydraulic conductivity [-]	ψ
n	porosity [-]	$\underline{x}$
S_s	specific storage [L^{-1}]	$\underline{x}$
z_b	elevation of the model lower boundary above datum [L]	x,y
z_g	elevation of the ground surface above datum [L]	x,y
z_{gs}	elevation of the stream channel bed above datum [L]	x,y[1]
z_r	elevation of the bottom of the root zone above datum [L]	x,y
z_{sp}	elevation of spring discharge point above datum [L]	x,y
$\Delta x,\Delta y,\Delta z$	dimensions of a column cell [L]	-
θ	volumetric soil water content [-]	ψ

[1] only defined for stream elements

Table 5.1 Model Parameters

variable		varies with	source[1]	non-zero range[2,3]
d_w	depth of ponded or stream water [L]	x, y, t	OC	-
e_s	evaporation rate at the ground surface [LT^{-1}]	x, y, t	ET	-
h_0	initial hydraulic head [L]	$\underline{x}$	ic	-
h_b	prescribed head at the model bottom boundary [L]	x, y, t	bc	-
h_l	prescribed head at the model boundary [L]	$\underline{x}$, t	bc	boundary only
q_b	rate of upflow from the deeper regional aquifer [LT^{-1}]	x, y, t	bc	-
q_l	prescribed lateral flux at the model boundary [LT^{-1}]	$\underline{x}$, t	bc	boundary only
q_p	net precipitation at the ground surface [LT^{-1}]	x, y, t	ET	-
q_t	specific volumetric sink term for plant extraction [T^{-1}]	$\underline{x}$, t	ET	$z_r \leq z \leq z_g$
q_w	specific volumetric flux out of an abstraction well [T^{-1}]	x, y, t	bc	well elements only
z_p	elevation of the top of the stream channel seepage face above datum [L]	x, y, t	OC[4]	channel only
z_s	elevation of stream water surface above datum [L]	x, y, t	OC	channel only

[1] ic = initial condition; bc = boundary condition or source/sink;
ET = evapotranspiration component; OC = overland/channel component.
[2] default non-zero range is the whole applicable model domain;
[3] z_r = elevation of bottom of root zone; z_g = elevation of ground surface (see Table 5.1);
[4] z_p is calculated within the VSS model using OC data

Table 5.2 Model Variables

5.3 Three-dimensional flow equation

The basic equation governing three-dimensional flow through a heterogeneous, anisotropic medium is

$$\eta\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial x}[K_x k_r\frac{\partial\psi}{\partial x}] + \frac{\partial}{\partial y}[K_y k_r\frac{\partial\psi}{\partial y}] + \frac{\partial}{\partial z}[K_z k_r\frac{\partial\psi}{\partial z}] + \frac{\partial(k_r K_z)}{\partial z} - q \qquad (5.2)$$

where the storage coefficient, η, is defined as

$$\eta = \frac{\theta S_s}{n} + \frac{d\theta}{d\psi} \qquad (5.3)$$

and q is a specific volumetric flow rate (volumetric flow rate per unit volume of medium) out of the medium, given by

$$q = q_w + q_{sp} + q_t \qquad (5.4)$$

where q_w accounts for well abstraction (section 5.4.6), q_{sp} accounts for spring discharges (Section 5.4.7), and q_t accounts for transpiration losses (Section 5.4.4).

The relationship between Θ and ψ is usually termed the water retention curve or the soil moisture characteristic (an example is given in Fig. 5.2). SHETRAN allows for any shape of a single-valued retention curve, and does not take hysteresis into account. A table of Θ and ψ values is required and intermediate values are then calculated, using a cubic spline method, and tabulated.

5.4 Initial and boundary conditions

5.4.1 Initial conditions

The initial conditions for a simulation are

$$h(\underline{x},0) = h_0(\underline{x}) \qquad (5.5)$$

where $h_0(\underline{x})$ is a prescribed hydraulic head field.

5.4.2 Stream-aquifer interaction

The exchange flows between an aquifer and a stream channel are composed of two parts: flow through the channel bed, and flow through the channel sides (including a seepage face).

Flow through the channel bed (which may be into or out of the channel) is handled in exactly the same way as flows at the ground surface, with a flowing stream channel corresponding to a ponded boundary condition at the ground surface (Section 5.4.5), and a dry channel corresponding to a flux boundary condition (Section 5.4.3). Low permeability sediments are handled by assigning appropriate values to the column cells at the stream bed.

For the channel sides, a pressure distribution is prescribed as a (time-varying) lateral head boundary condition, given by

$$h = \begin{cases} z_s & z_{gs} < z < z_s \\ z & z_s < z < z_p \end{cases} . \tag{5.6}$$

The specific volumetric flow rate, q_{ex}, into the stream channel through the channel sides is then given by

$$q_{ex} = K_{ex}^{eff} \, k_r \, \Delta h \tag{5.7}$$

where K_{ex}^{eff} is the effective hydraulic conductivity, defined as the weighted harmonic mean of the conductivities in the adjacent element and in the stream sediments, given by

$$K_{ex}^{eff} = \frac{(d_x + d_B) K_a K_B}{d_x K_B + d_B K_a} \tag{5.8}$$

where d_x is the distance from the stream channel side to the adjacent computational node, and K_a is the saturated hydraulic conductivity of the aquifer (= K_x or K_y, depending upon the channel orientation).

5.4.3 Precipitation/evaporation

During periods of the simulation when no ponded water exists at the ground surface, the flux boundary condition is given by

$$k_r K_z \left(\frac{\partial \psi}{\partial z} + 1\right) = q_p - e_s \quad . \tag{5.9}$$

5.4.4 Plant transpiration

Plant transpiration rates are controlled by a driving potential governed by atmospheric conditions, a resistance of the vegetation material to the passage of water, and the availability of soil water. In SHETRAN, the transpiration rates are calculated in the evapotransiration/interception (ET) component, and are extracted from the subsurface as a sink term, q_t.

5.4.5 Interaction with ponded ground surface water

During periods of the simulation when ponded water exists at the ground surface (including stream water for stream elements), the head boundary condition is given by

$$\psi|_{z=z_g} = d_w \quad . \tag{5.10}$$

The flux through the ground surface (negative for infiltration, positive for exfiltration) is then

$$I = - K_z \left(\frac{\partial \psi}{\partial z} + 1\right) \quad . \tag{5.11}$$

5.4.6 Well abstraction

For a well whose screen intersects n_c cells, the specific volumetric rate of flow from each cell, q_{wi}, is given by

$$q_{wi} = \frac{q_w \overline{K_i} \Delta z_i}{\sum_{j=1}^{n_c} \overline{K_j} \Delta z_j} \tag{5.12}$$

where $_i$ is the mean saturated lateral hydraulic conductivity of cell *i*.

5.4.7 Springs

In the VSS model it is assumed that water discharging from a spring flows into a stream channel (this may be into the top of a stream channel, or into a channel reach if the spring discharge point is located along a valley side). The specific volumetric spring discharge, q_{sp}, is given by

Install Equation Editor and double-click here to view equation. (5.13)

where ψ_c is the pressure potential at the aquifer cell from which discharge takes place, and z_c is the elevation of the cell node (above datum).

5.4.8 Bottom boundary conditions

The lower boundary of the model should normally be located at a depth where substantial flows do not occur over the timescale of a simulation. To allow a small contribution of a deeper regional aquifer to the modelled region, a prescribed rate of inflow can be defined along the lower boundary by

$$- k_r K_z \left(\frac{\partial \psi}{\partial z} + 1 \right) = q_b \qquad (5.14)$$

where the regional aquifer flow, q_b, is positive into the model. The default case is for the lower boundary to be an impermeable bed, so that $q_b = 0$.

To allow for the possible use of the model to simulate freely draining soil in a column, a free drainage flux lower boundary condition is included, given as a unit head gradient

$$\frac{\partial h}{\partial z} \Big|_{z=z_b} = 1 \ . \qquad (5.15)$$

Alternatively, a prescribed time-varying spatially-varying head boundary condition can be defined, as

$$h \big|_{z=z_b} = h_b(x, y, t) \ . \qquad (5.16)$$

5.4.9 Lateral boundary conditions

For some uses of the model, prescribed boundary conditions need to be given at the edge boundaries of a region, where a full catchment is not being modelled. These may be required, for example, in simulating experimental plots, sub-regions of a catchment (to define a locally finer grid for contaminant simulations), or catchments where the groundwater divide does not coincide with the surface water divide.

A head boundary condition can be defined along a part of the model boundary B_1, given by

$$h(\underline{x}, t) \quad = \quad h_l(\underline{x}, t) \quad \text{for } \underline{x} \text{ on } B_l \ . \tag{5.17}$$

A flux boundary condition can be defined along a part of the model boundary B_2 as

$$\underline{n}.(k_r \underline{\underline{K}}.\Delta h) \quad = \quad q_l \tag{5.18}$$

where $\underline{n}$ is the unit outward normal to the model boundary.

Note that, to maintain full flexibility, flux boundary conditions are allowed both into and out of the modelled region. No feedback is provided for specified fluxes out of the model (which could lead to unphysical dewatering of the aquifer), so particular care should be taken if this type of boundary condition is used.

6 Snowmelt Module

6.1 Introduction

The snowmelt (SM) module calculates:

- snowpack depths;
- snowpack temperatures;
- snowmelt rates;
- interception of snowfall by vegetation;
- sublimation from the snowpack.

In the basic model the processes are modelled as follows. When rain falls, a proportion is intercepted by the vegetation canopy, a proportion falls through the canopy onto the snowpack (if a snowpack exists) and a proportion falls directly on the snowpack covering ground where there is no vegetation. If the air temperature falls below 0°C, any previously intercepted rain remains on the canopy and there is no drainage, ie the intercepted water is assumed to be frozen and there is no evapotranspiration. If the air temperature remains above 0°C, intercepted rain drains from the canopy and evapotranspiration can occur. When snow falls, there is no interception and all the snowfall is added to the snowpack. Melting of the snowpack may occur and the resulting meltwater and any rainfall trickle through the snowpack, eventually reaching the soil surface. This combined meltwater and rainfall delivery replaces the net precipitation at the soil surface referred to in the ET module documentation of Section 3 (see Fig. 6.1).

There are two methods available for calculating snowmelt, the degree day method and the energy budget method.

6.2 Interception

The purpose of the interception model is as described in the ET module documentation, Section 3.3, namely to calculate net precipitation below the canopy, amount of water stored on the canopy and evaporation from the canopy.

Different processes are assumed to be in operation depending on whether the air temperature is above 0°C or not. The processes are also affected by the degree to which the vegetation may be buried by snow.

6.2.1 Air temperature ≤ 0ºC

It is assumed that all precipitation is in the form of snow and that all the snow passes directly to the snowpack (or ground if there is no snowpack) without any interception.

This is an approximation since snow can accumulate to large depths on vegetation during a snowstorm, generally falling off or being blown off subsequently. However, the error involved in routing the snowfall directly to the snowpack is probably acceptable since: (a) for a thick snowpack, the delivery of meltwater to the ground is not greatly affected by a delay of two or three days in the supply of intercepted snow to the top of the snowpack, so therefore the delay can be neglected; and (b) if the snowpack is thin and melting occurs, the intercepted snow (which cannot be deeper than the snowpack) is also melting and contributing to the meltwater of the snowpack so the delivery to the ground should be similar to that which would result from a thin snowpack composed of all the snowfall, again allowing the interception process to be neglected.

6.2.2 Air temperature > 0ºC

It is assumed that all precipitation is in the form of rainfall and that its interception,. storage, evaporation and drainage are described by the Rutter model (Section 3). However, if the air temperature falls below 0°C while there is still intercepted water on the canopy, that water is assumed to freeze so that there is no drainage or evaporation.

Evaporation of intercepted water is assumed to occur only from the total leaf area above the snowpack. As noted in Section 3.3.1, when the depth of water, C, on the canopy equals or exceeds the storage capacity, S, the evaporation is assumed to occur at the potential rate, E_p. When C is less than S the rate is assumed to be E_p C/S.

6.2.3 Snowpack depth exceeds vegetation height

This situation is unlikely to apply to trees but is important for grass, scrub and other short vegetation. As the canopy is covered there is no canopy interception and all the precipitation is added to the snowpack. Evaporation or sublimation from the snowpack is modelled only in the energy budget method.

6.3 Evapotranspiration

It is assumed quite simply that, if the air temperature is less than 0°C, all moisture is frozen and there is no evapotranspiration. However, sublimation from the snowpack can be modelled using the energy budget method.

If the air temperature is above freezing, evapotranspiration from the canopy is modelled is calculated in the ET module (unless the canopy is entirely covered by snowpack). However, as long as a snowpack exists, soil evaporation is assumed to be zero.

6.4 Snowmelt

The purpose of the snowmelt module is to model the snowpack thickness as it is affected by precipitation and melting, and to model the rate of delivery of meltwater from the snowpack to the soil surface. Snowmelt can be modelled using either the degree day method or the energy budget method. In each case, vertical variations in snowpack parameters are neglected and conditions are assumed to be uniform with depth.

6.4.1 Snowmelt by degree-day method

This method relies on the fact that, of all the meteorological variables, air temperature is usually the most highly correlated with snowmelt rate (Zuzel and Cox, 1975). In the traditional form of the method, the approximate snowmelt is calculated directly as (US Army Corps of Engineers, 1956)

$$M = K * (T_a - T_0) * SG \qquad 6.1$$

Where M = melt rate; K = the degree-day factor; Ta = air temperature; T0 = the melting point temperature in ºC; and SG = specific gravity of snow. T0 is often set to 0 ºC so that

$$M = K * T_a * SG \qquad 6.1$$

And if the air temperature is less than 0 ºC there is no melting.

6.4.2 Heat flux by energy budget method

This method is intended for use when full meteorological data are available. Total heat flux to the snow is derived by calculating the contributions from all component fluxes for the pack as a whole (Fig. 6.2). At the upper boundary of the pack these are (in units of energy per unit time per unit area): R_n the net radiation; C, the heat gained by convection from the air; I, the heat gained from precipitation; and V, the heat gained from condensed vapour. V is negative when the snow melts and evaporates or sublimes directly into the atmosphere. The inputs of energy per unit time per unit area at the lower boundary are: G, the heat conducted from the underlying rock or soil; and MH, the latent heat lost through movement of water out of the pack. The fluxes are calculated by the following methods.

Heat input from net radiation is given by meteorological data. Heat input from the ground is assumed to be constant at 2 J/m^2/s.

Heat gained from precipitation depends on whether the precipitation is rain (air temperature > 0°C) or snow (air temperature ≤ 0°C). For rain

$$I = \rho_w * P * (c_p)_w * T_a \qquad 6.3$$

And for snow

$$I = \rho_w * P * (c_p)_i * [T_a - \check{T}] \qquad 6.4$$

Where P = precipitation rate (in water equivalent); $(c_p)_w$ = specific heat of water at constat pressure; $(c_p)_{i\,=}$ specific heat of ice at constat pressure; and $\check{T}$ = the calculated average temperature of the snowpack. In each case the falling precipitation is assumed to be at air temperature T_a. For Eq. 6.3, it is assumed that T_a is in °C, the melting point is 0°C and that, on reaching the snowpack, the rain is cooled to 0°C but remains liquid and joins the snowmelt

Heat input from convection (ie sensible heat transferred by turbulent exchange from the atmosphere) may be estimated as

$$C = \rho_a (c_p)_a D_h (T_a - T_s) \qquad 6.5$$

Where ρ_a = the density and $(c_p)_a$ = specific heat of air; T_s = the temperature of the snow surface; and = D_h a turbulent transfer coefficient. Similarly the evaporation or sublimation rate may be estimated as

$$E = \rho_a D_h [q_s(T_s) - q_a] \qquad 6.6$$

Where $q_s(T_s)$ = specific humidity of the snow surface which is saturated at temperature T_s; and q_a = humidity of the atmosphere. In the energy budget method the temperature T, is not known and must therefore be replaced in Eqs. 6.5 and 6.6 by $\check{T}$, the calculated average temperature of the snowpack. It is also assumed that the turbulent transfer coefficient for water vapour is the same as that for sensible heat. The heat flux from this phase change (condensation or evaporation of vapour) is given by where L_{vw} = latent heat of vaporisation of water.

$$V = [L_{vw} + L_{wi} - (c_p)_i * \check{T}] * E \qquad 6.7$$

Finally the total heat flux HFT (apart from the contribution MH) is calculated as

$$HFT = C + I + V + G + R_n \qquad 6.8$$

For neutral conditions in the boundary air layer (no gradient in air temperature from the snow surface) the turbulent transfer coefficient D_h used in Eqs. 6.5 and 6.6 is taken to be equal to the momentum transfer coefficient obtained from the logarithmic wind velocity profile, i.e.

$$D_h = \frac{\kappa^2 u}{\left[\ln\left(\frac{z' - Z - d}{z_o}\right)\right]^2} \qquad 6.9$$

where κ = von Karman's constant (= 0.41); u = wind speed, measured at z′, the height of the anemometer above the solid ground surface; Z = snowpack thickness at the anemometer site; d = zero plane displacement; and z_o = the aerodynamic roughness of the snow surface.

For stable or unstable conditions, a correction to Eq. 6.9 is implemented, following a method by Price and Dunne (1976). For stable conditions ($T_a > T_s$), D_h is replaced by

$$(D_h)_s = \frac{(D_h)_n}{(1 + 10\ Ri)} \tag{6.10}$$

and for unstable conditions ($T_a \leq T_s$), D_h is replaced by

$$(D_h)_u = (D_h)_n\ (1 - 10\ R_i) \tag{6.11}$$

Where $(D_h)_n$) is the value of D_h for neutral conditions (Eq. 6.9).

The Richardson number is

$$Ri = g\ (z' - Z - d) \left(\frac{T_a - T_s}{u^2} \right) \frac{1}{(T_a + 273)} \tag{6.12}$$

where g = acceleration due to gravity and the temperatures are in °C.

Meteorological data used in the energy budget method consist of: precipitation, P; wind speed, u; air temperature, T_a; specific humidity, q_a; and net radiation, R_n; all measured at a height z' above the ground.

6.4.3 Snowmelt by energy budget method

Once the total heat flux HFT has been calculated by 6.8 the resulting snowmelt (if any) can be derived. First an energy balance equation

$$HFT - MH = (c_p)_i \frac{\partial \bar{T}}{\partial t} \bar{\rho}_s Z \tag{6.13}$$

is defined for the snow pack as a whole, where the right hand term of the equation represents the change in the heat content of the pack. Ť = average temperature of the snow; ρ_s = average density of the snow in the pack; Z = snowpack thickness; t = time; and MH = energy available to melt snow. Then, Eq. 6.13 is used to find out whether the input of energy is sufficient to raise the snowpack temperature to above 0°C. If it is, any energy in excess of the amount required to produce that state is directed to produce snowmelt.

In the method the average snowpack temperature is constrained by the inequality Ť ≤ 0°C. For melting to occur, the snowpack must be isothermal at 0°C: there can then be no change in heat storage, so the right hand side of Eq. 6.13 becomes zero. When the snowpack temperature drops below 0°C, there is no melting, so MH becomes zero. Thus either MH or the right hand side of Eq. 6.13 must be zero at all times. It is also assumed that the only movement of water across the lower boundary is that of water leaving a melting pack.

As a first step in the calculations, it is assumed that MH = 0 and, knowing an average value of HFT over a time period Δt (from time t to t+ 1), the average temperature at time t+ 1 is calculated from the temperature and depth of the pack at time t, based on the hypothesis that all the heat flux is directed towards changing the snowpack temperature and not towards causing melting. Then, via Eq. 6.13

$$\bar{T}^{t+1} = \bar{T}^{t} + \frac{\Delta t}{(c_p)_i \, \bar{\rho}_s \, Z^t} * HFT \qquad 6.14$$

where Z^t = depth of the pack at time t.

If this calculation produces a value of $\check{T}^{t+1}$ which is less than 0°C, the assumption that MH = 0 is correct. There is no snowmelt and the hypothesis that the heat flux contributes only to a temperature change is also correct. However, should the calculation produce a value of $\check{T}^{t+1}$ which is greater than 0°C (which is physically unrealistic) the assumption that MH = 0 is false. Then, as a second step, the heat flux has to be divided into the amount which brings the snowpack temperature up to 0°C and the remainder which is devoted to melting. MH is calculated from Eq. 7.13 with $\check{T}^{t+1}$ set to 0°C

$$MH = HFT - \left[- (c_p)_i \, \bar{\rho}_s \, \frac{Z^t \, \bar{T}^t}{\Delta t} \right] \qquad 6.15$$

Melting occurs and the loss of water from the pack per unit area per unit time (i.e. the depth rate of loss) is

$$M = \frac{MH}{\rho_w L_{wi}} \qquad 6.16$$

where ρ_w = the density of water; and L_{wi} = latent heat of fusion of ice. Whether or not there is enough snow to satisfy this rate depends of course on the thickness of the snowpack and any losses due to sublimation (which are catered for before those due to melting).

6.4.4 Meltwater routing

Melting is assumed to occur at the top of the snowpack rather than at the bottom (heat input from the ground being of minor influence), so the meltwater, together with any rainfall, has to be routed through the snowpack. The time which it takes to trickle through the snowpack is calculated using the equation (Anderson, 1968).

$$t_m = 0.745 \, Z^2 + 1.429 \, Z \qquad 6.17$$

where t_m = the time of travel from the top to the bottom of the snowpack in hours; and Z = snowpack thickness at the beginning of the passage of the meltwater, in metres of snow. Eq. 6.17 was derived empirically from experimental data. There is no facility for refreezing or absorption of meltwater in the snowpack and all rainfall is equally assumed to remain in the fluid state and to be routed through the pack without loss.

6.4.5 Snowpack thickness

If the air temperature does not exceed 0°C, any precipitation is in the form of snow. This is added directly to the snowpack. If the air temperature is above 0°C, precipitation is in the form of rain and is added directly to the meltwater in the snowpack, i.e. it does not affect snowpack depth. All rainfall reaching the snowpack is net rainfall after interception in the canopy (unless the canopy is buried by the snowpack).

The snowpack thickness can increase only as a result of snowfall and can decrease only as a result of melting or sublimation. No allowance is made for settling of the snow or effects due to ageing.

6.5 Numerical algorithms

The algorithms for the SM module are straight forward, involving the calculation of snowpack depth, heat budget for the snowpack, and snowmelt, for each element individually. The sequence of calculations is as follows.

1. If the air temperature, Ta, is less than zero, add net precipitation to the snowpack as snowfall.
2. For the degree-day method, calculate snowmelt.
3. For the energy budget method, calculate heat input to the snowpack, and the resulting change in the temperature of the snowpack. If the new temperature is greater than zero, set the temperature to zero and calculate snowmelt resulting from excess heat.
4. Calculate the new snowpack depth.
5. Calculate the time for any new meltwater slugs to move through the snowpack.
6. Calculate if any meltwater slugs from previous timesteps have reached the bottom of the snowpack, and add into net snowmelt at the bottom of the snowpack.

7 Bank element module

The bank element (BK) module exists only to establish data for bank elements. All numerical calculations for bank elements take place within the other SHETRAN flow modules.

The subsurface flow exchanges between a bank element and its adjacent elements are shown in Fig. 9.1. The element is represented by an L-shaped cross-section for subsurface flows, where the effective channel cross-section (the dotted line in Fig. 9.1) taken out of the bank element has the same area as the actual channel cross-section.

The computation for each hydrological process takes place within the relevant SHETRAN flow modules. Lateral saturated zone flows between the bank element and the adjacent grid element (Q_{S2}) and beneath the channel (Q_{S1}) are computed in the variably saturated subsurface module. Exchange flows between the saturated zone and the channel through the channel bed (Q_B) and the channel face (Q_F) are also computed in the variably saturated subsurface module. Recharge to the saturated part of the aquifer (Q_R) is part of the variably saturated subsurface module. If the channel link is dry, infiltration into the channel bed (Q_I) is computed in the evapotranspiration module as precipitation minus evaporation (based on the moisture content of the unsaturated cell adjacent to the channel bed). In this case, Q_I is transferred to the variably saturated subsurface module as a source term in the flow equation.

A further flow is calculated for bank elements (compared with grid elements) to allow for the direct extraction of channel water by plants in the bank elements (Q_E). This is computed in the evapotranspiration module, based on an extension to the root density function for bank elements only.

8 References

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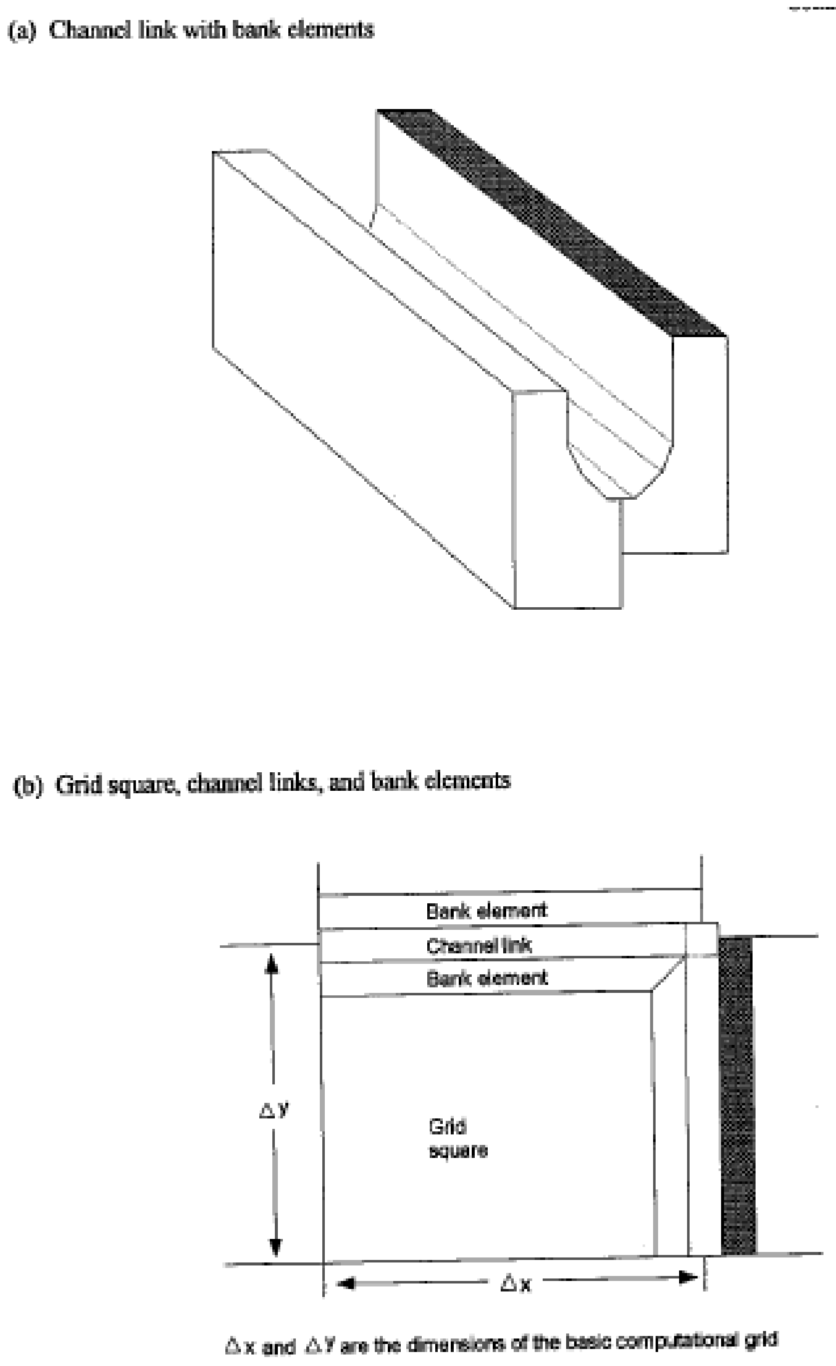


Figure 2.1 Computational elements in SHETRAN

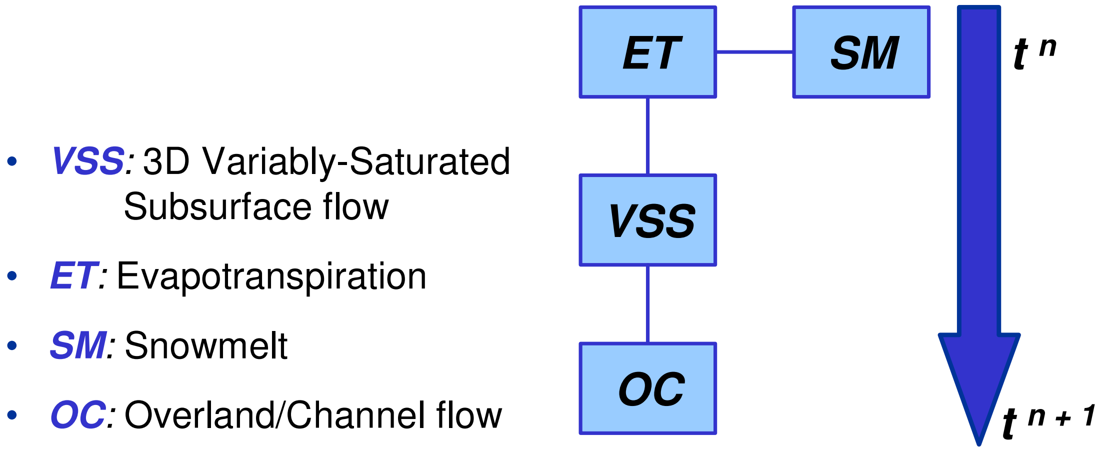
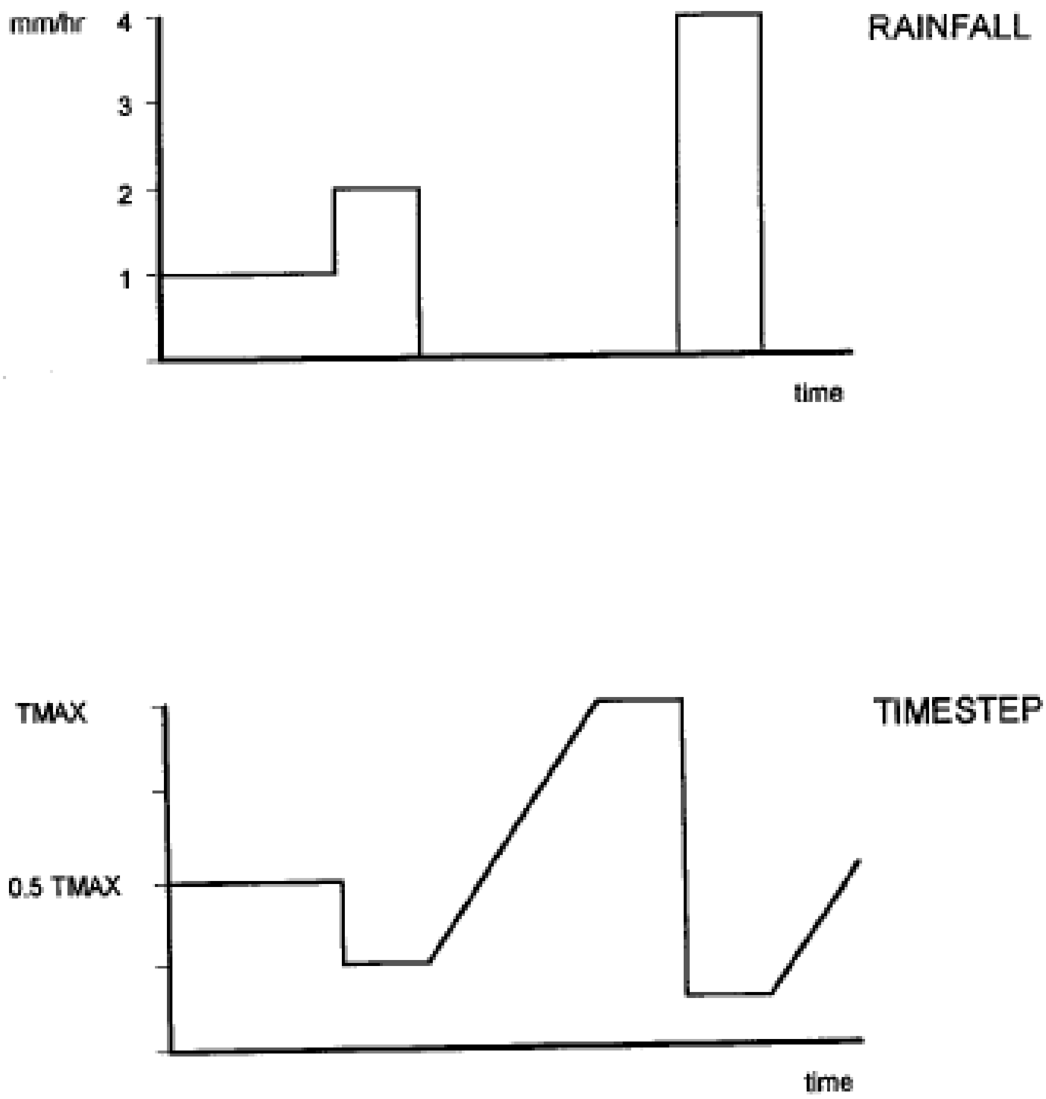


Figure 2.2 Order of execution of components and modules. NB The BK module is integrated into each of the flow modules



Basic timestep = TMAX

Maximum rainfall allowed per timestep = 0.5 mm

Figure 2.3 Example of timestep calculations

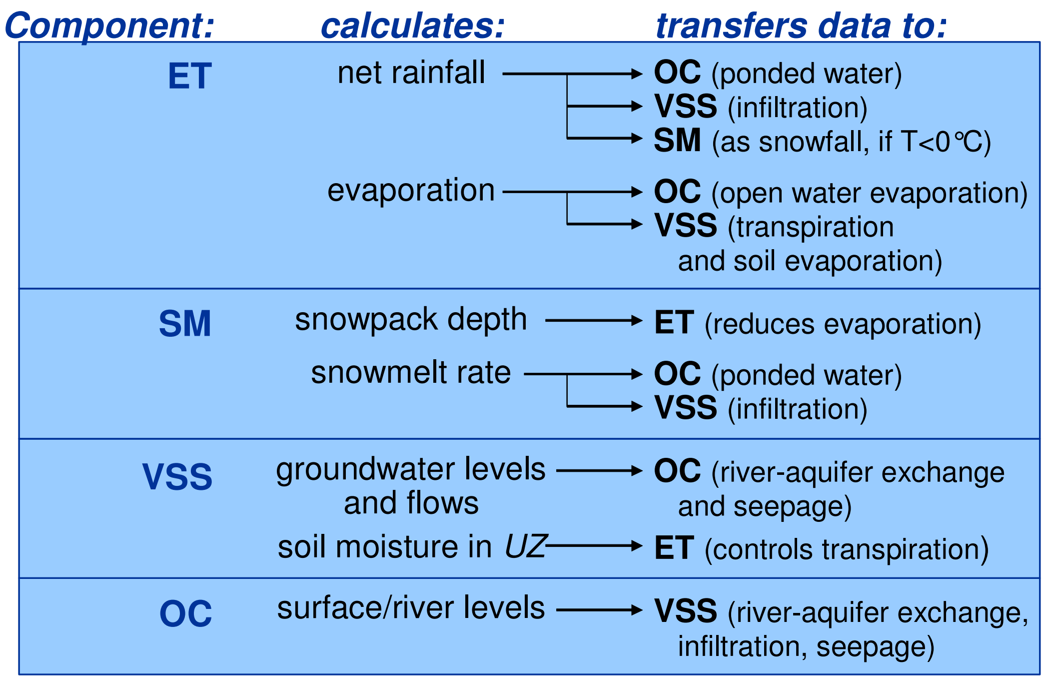


Figure 2.4 Data exchange between flow modules

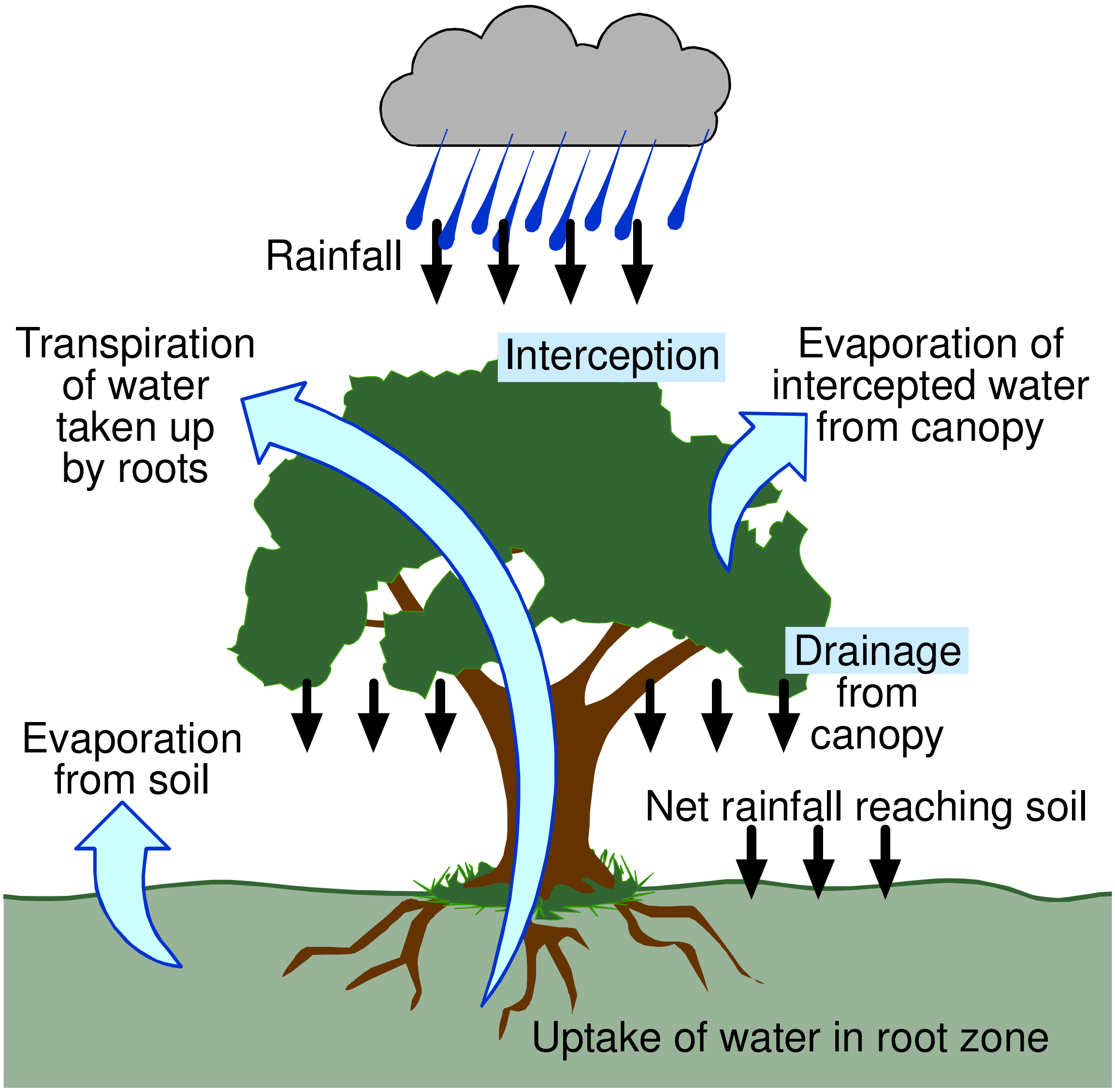


Figure 3.1 Processes modelling in the evapotranspiration/interception module

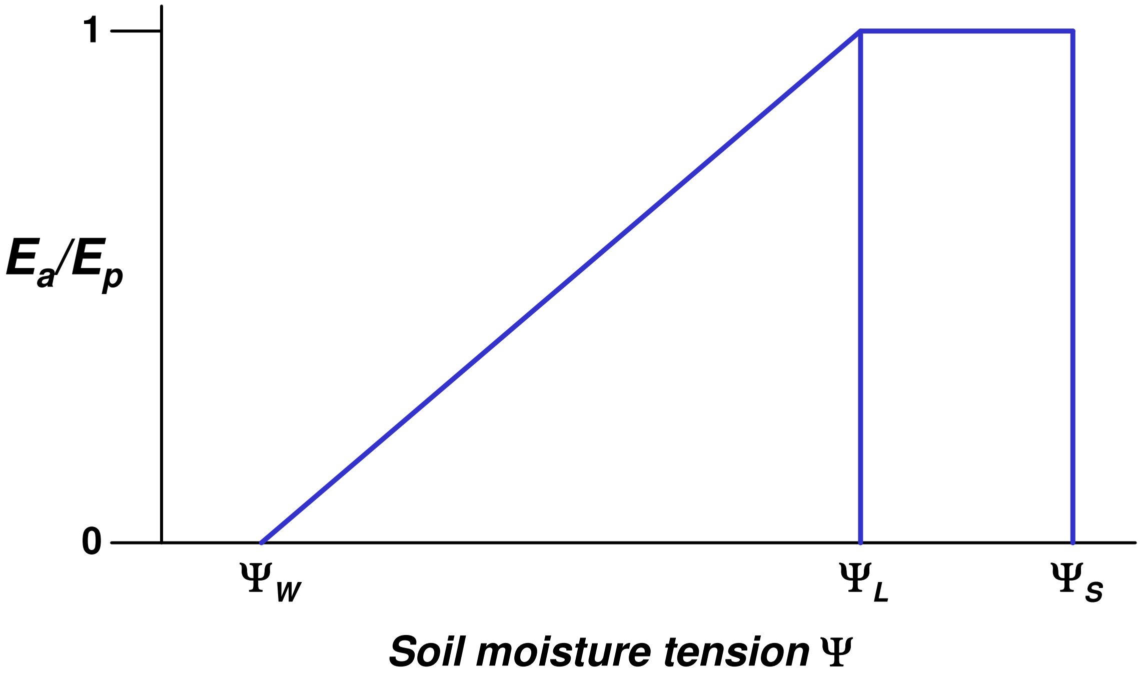


Figure 3.2 Dependency of ration of actual to potential evapotranspiration on soil moisture tension (Feddes et al. 1976)

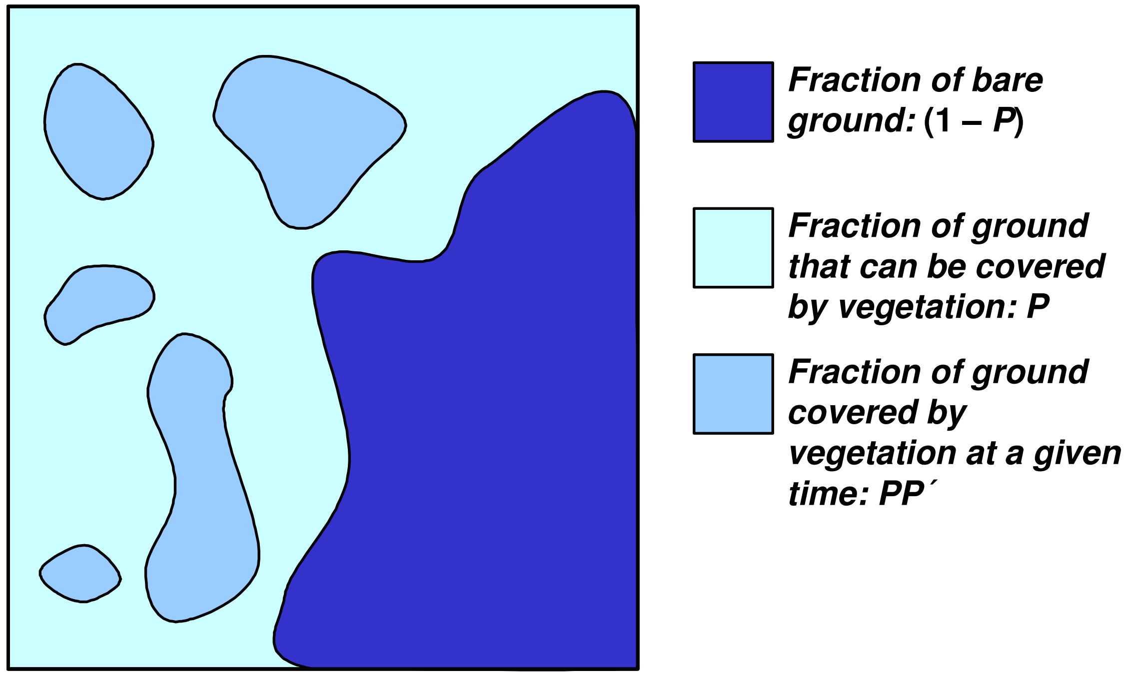


Figure 3.3 Definition diagram for P and P′

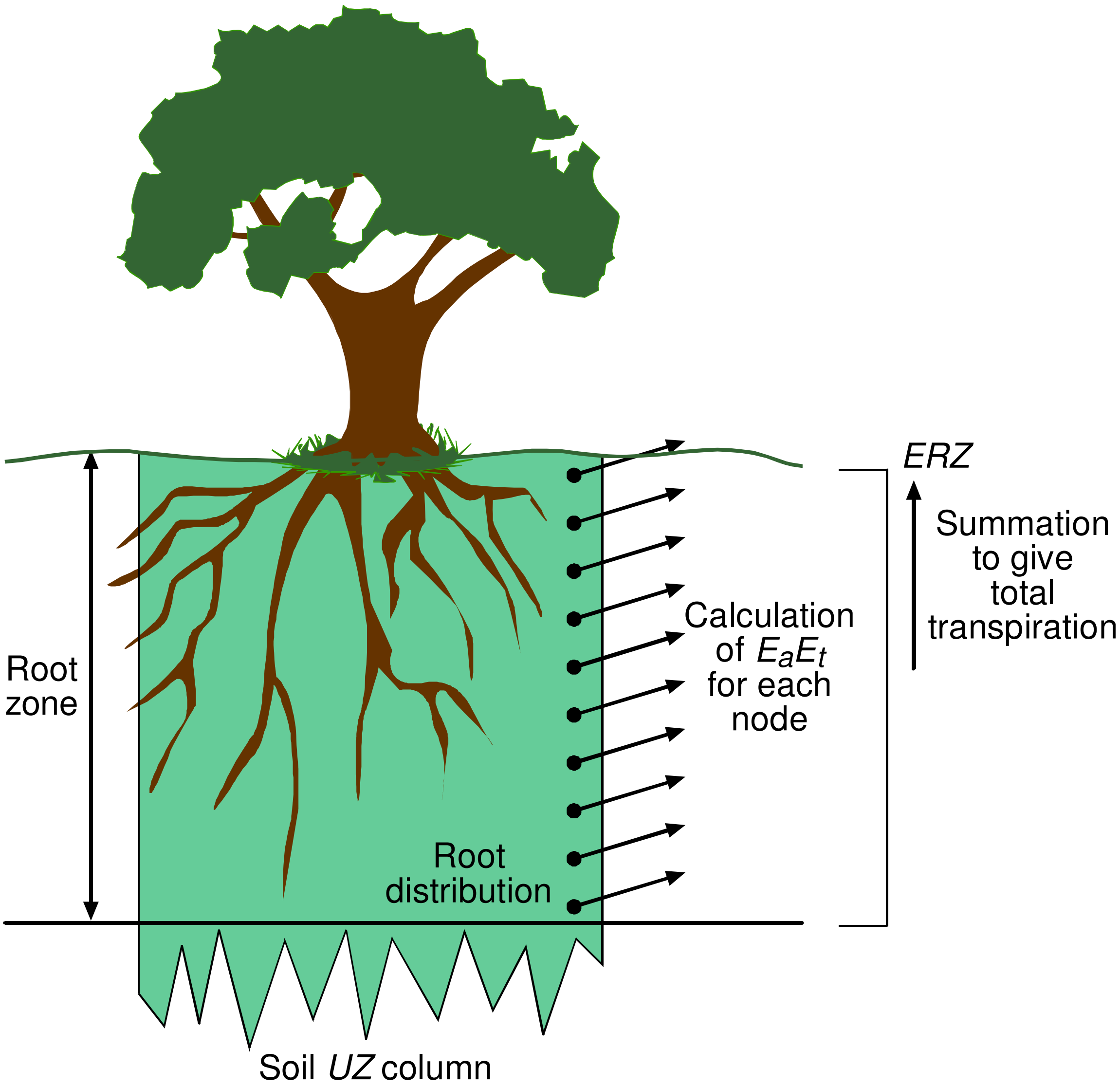
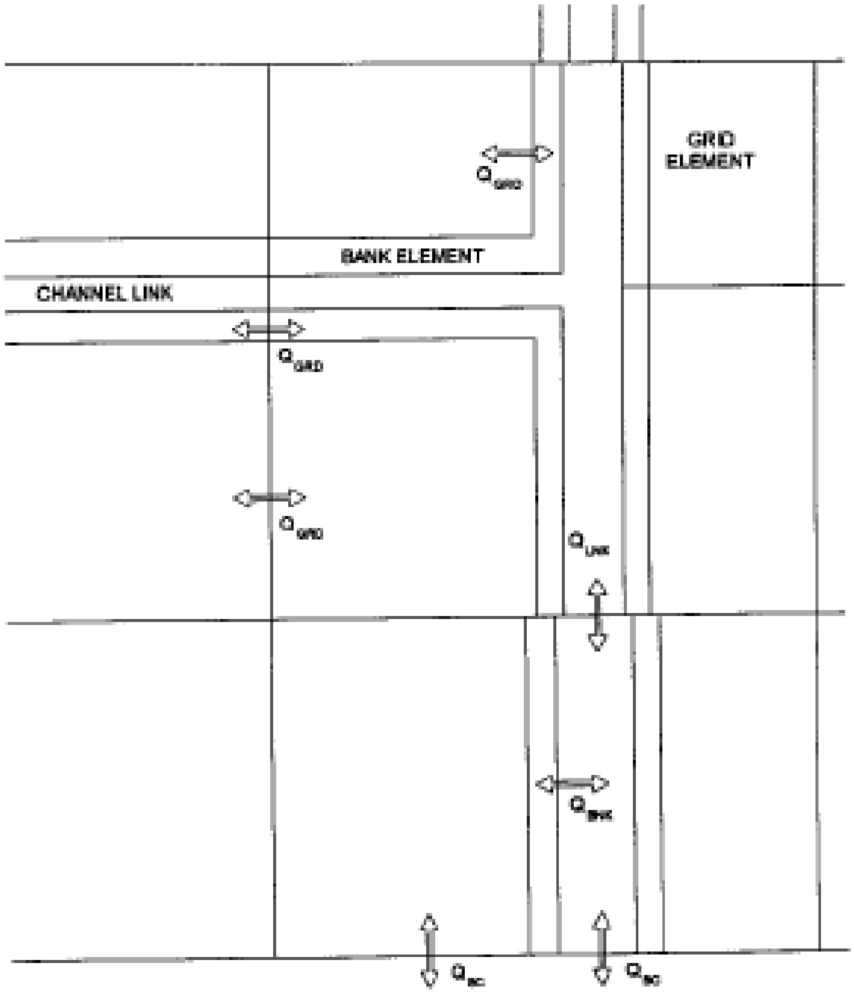


Figure 3.4 Diagrammatic representation of the transpiration model



Q_{GRD} Manning resistance flow equation between two grid or bank elements

Q_{BNK} Manning resistance flow equation for inflow from a grid or bank element into a channel link; or flat-crested weir equation for overbank flooding

Q_{LNK} Manning resistance flow equation between two channel links of varying cross-section

Q_{BC} Flux boundary conditions:
(a) Flat-crested weir (channel only)
(b) Prescribed flow rate
(c) Flow is a polynomial function of water depth

Figure 4.1 Overland and channel flows

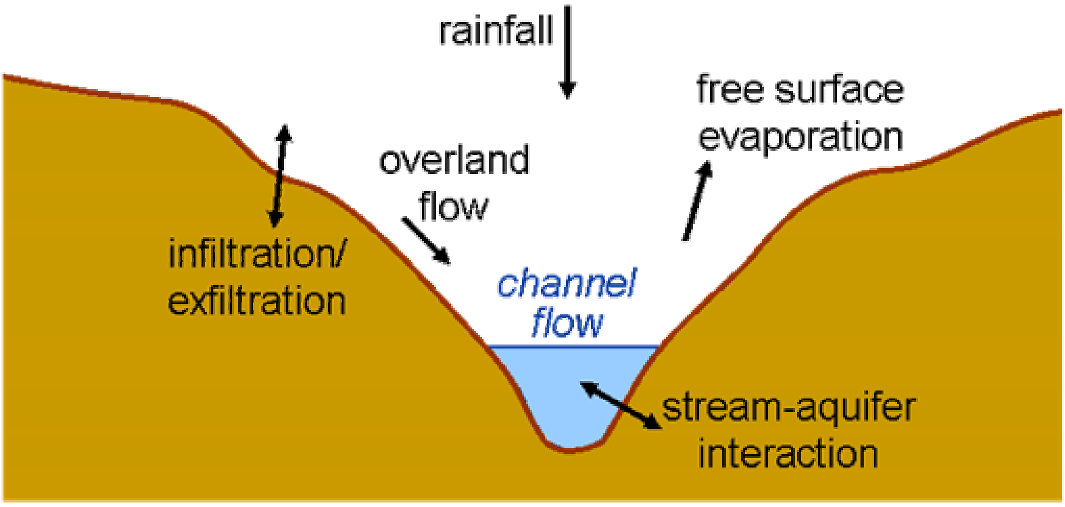


Figure 4.2 Processes modelled in the Overland flow / Channel module

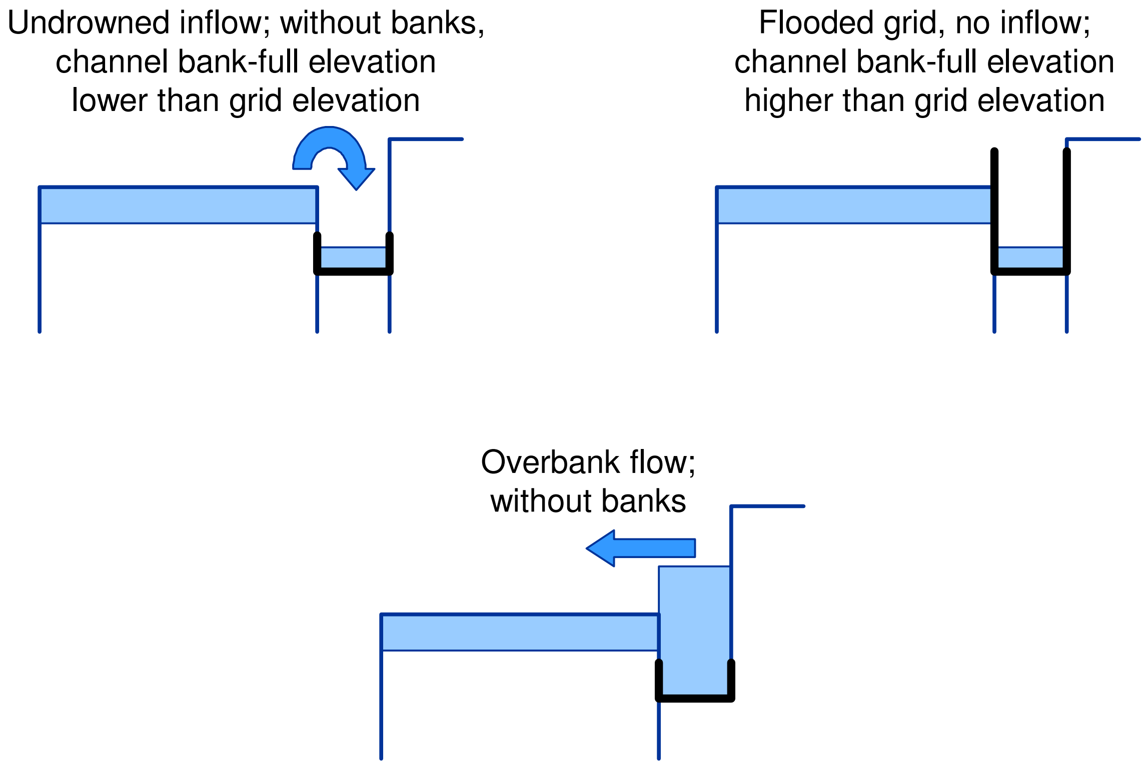


Figure 4.3 Examples of flow configurations between grid/bank and channel

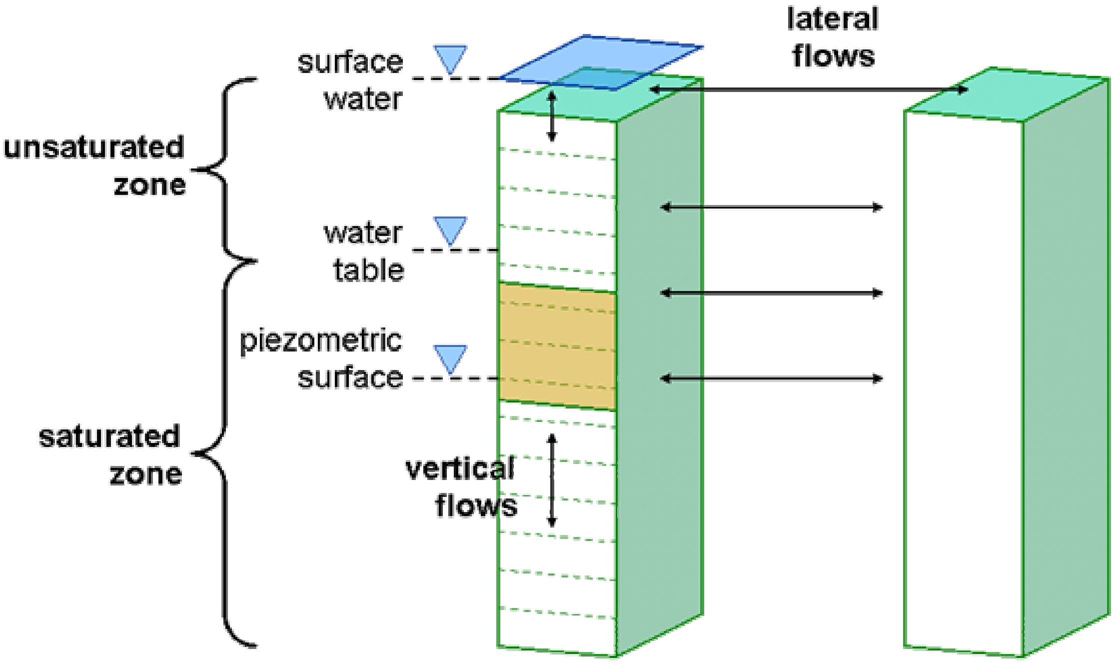


Figure 5.1 Processes modelled in the variably saturated subsurface module

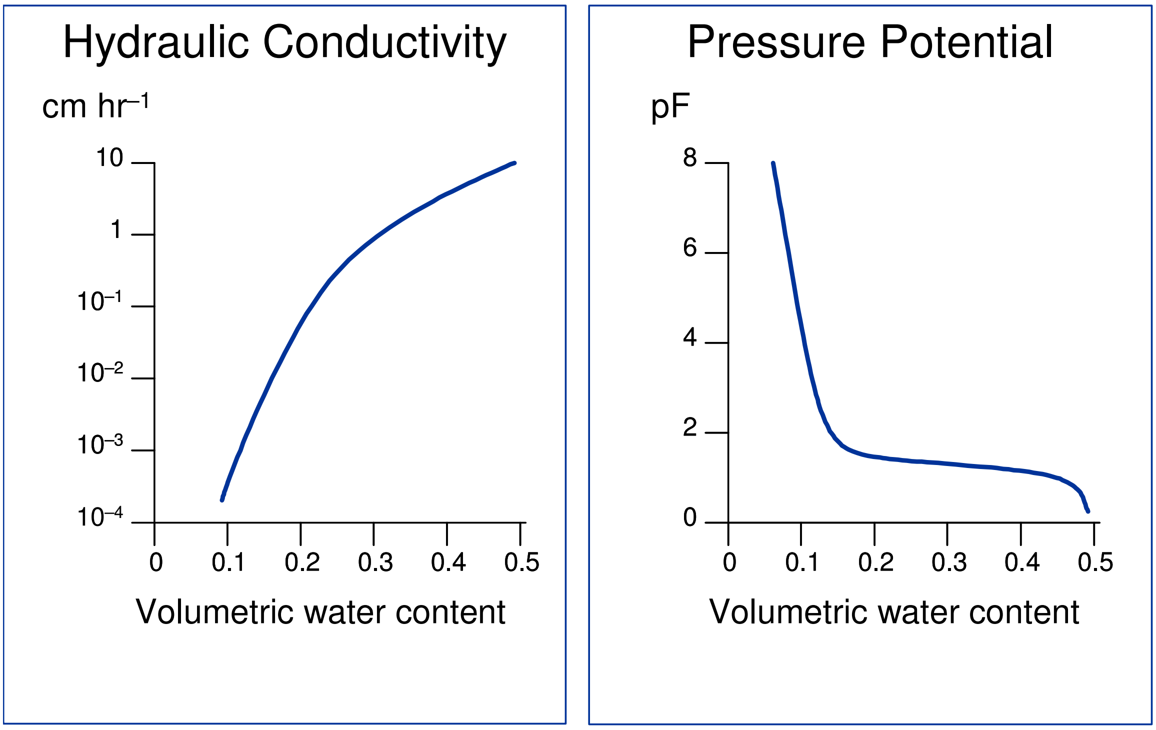


Figure 5.2 Example of soil characteristics for variably saturated subsurface module

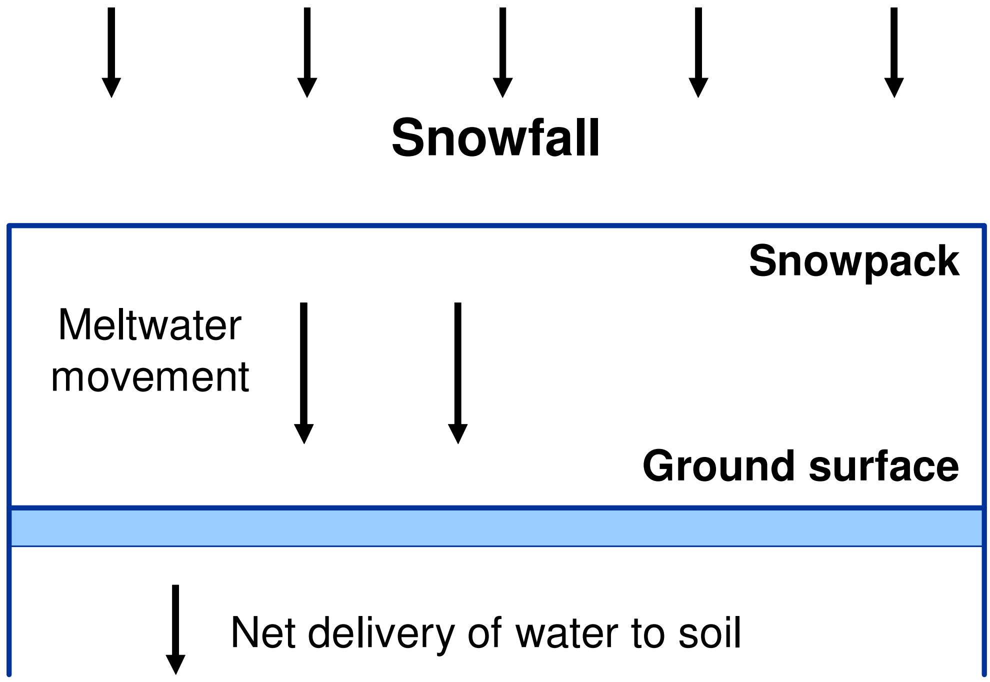


Figure 6.1 Processes modelling in the snowmelt module for air temperatures not exceeding 0ºC

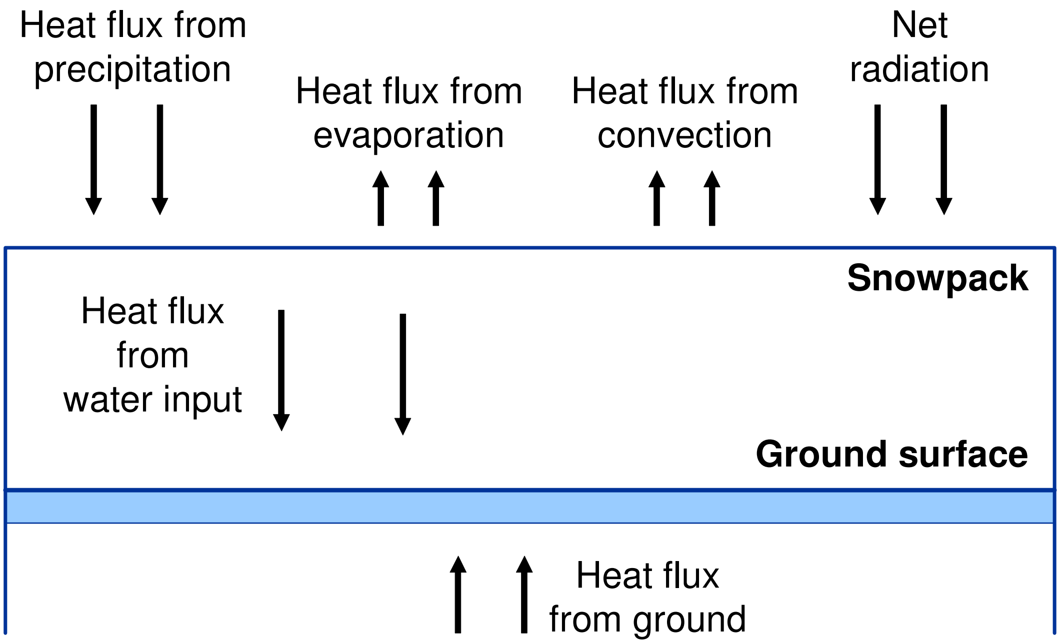
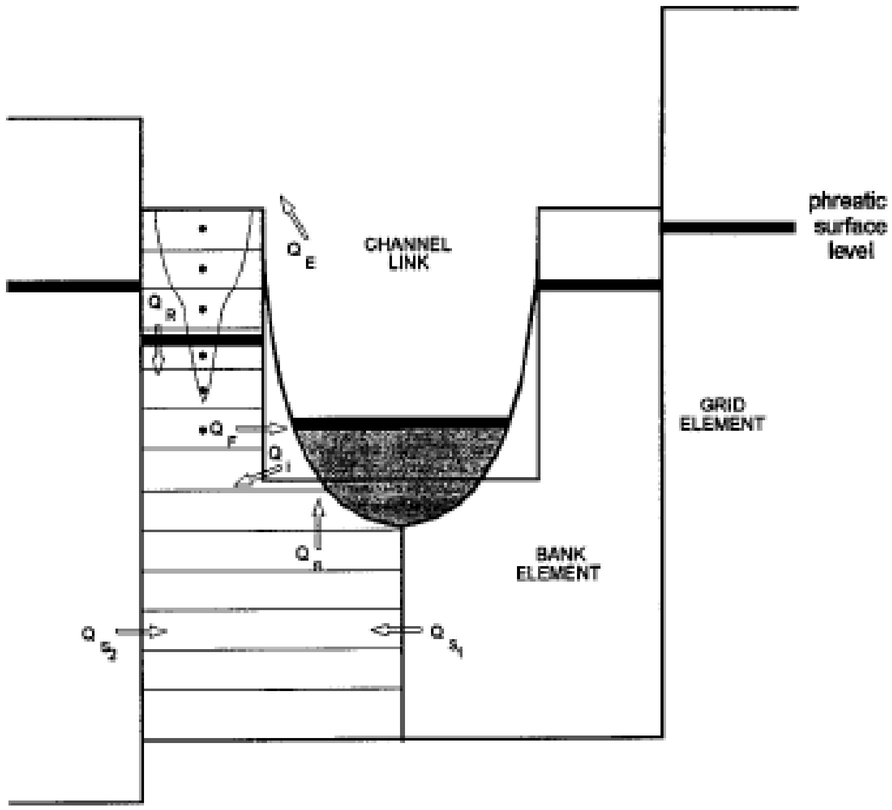


Figure 6.2 Energy fluxes to the snowpack considered in the energy budget model



Q_{S_1}	Saturated zone flow beneath channel
Q_{S_2}	Saturated zone flow from adjacent grid element
Q_B	Exchange flow through (effective) channel bed
Q_F	Exchange flow through (effective) channel face
Q_I	Infiltration into dry channel bed (precipitation minus evaporation)
Q_R	Recharge from unsaturated zone
Q_E	Direct plant uptake from channel water

NB Ground surface or water surface fluxes not shown

Figure 7.1 Bank element and channel link flow exchanges