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# SHETRAN sediment transport component: equations and algorithms

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**SIMULATING RADIONUCLIDE TRANSPORT FROM THE GEOSPHERE TO THE BIOSPHERE**

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AUTHOR(S) AND AFFILIATION: Lukey B.T., Bathurst J.C., Hiley, R.A. and Ewen, J.

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| | NAME | SIGNATURE | DATE |
|---|---|---|---|
| AUTHOR(S) | Ben Lukey | Ben Lukey | 1/11/95 |
| CHECKED | JAMES C BATHURST | James Bathurst | 1/11/95 |
| APPROVED | JOHN EWEN (Senior Officer) | John Ewen | 1/11/95 |
| APPROVED FOR EXTERNAL DISTRIBUTION | JOHN EWEN (Principal Investigator) | John Ewen | 1/11/95 |

Water Resource Systems Research Unit
Department of Civil Engineering
University of Newcastle upon Tyne
Newcastle upon Tyne NE1 7RU

Tel: Tyneside (0191) 222 6259
Fax: Tyneside (0191) 222 6669

## Executive Summary

This document presents the equations and algorithms used by the SHETRAN sediment transport component. The erosion processes simulated by the component are soil erosion by raindrop and leaf drip impact, detachment of soil by overland flow and channel erosion. Sediment is transported by overland and channel flows calculated in the flow component of SHETRAN, and is routed through the catchment using the sediment continuity equation.

**Contents** **page**

# 1 SHETRAN SEDIMENT TRANSPORT COMPONENT

This document presents the process and routing equations for the SHETRAN sediment transport component. The component simulates the erosion, transport and deposition of sediments at the ground surface and along the channel network and is divided into two subcomponents: hillslope and channel. The hillslope subcomponent represents the erosion and transport of soil at the ground surface (including the surface of the SHETRAN channel bank elements); it allows for a redistribution of soil across the ground surface of the catchment and an input of sediment to the channel system. The channel subcomponent represents the transport of sediment along the channel network, allowing for supply from outside the channel, for in-channel erosion and deposition and for overbank deposition.

The component is driven by the time varying flow characteristics (discharge, velocity, depth) determined by the SHETRAN flow component. There is no feedback in terms of sediment concentration or altered geomorphology to the flow component but details of the sediment load form an input to the contaminant migration component. Many of the model parameters required by the sediment component are already provided for the flow component, but a few require separate provision.

SHESED-UK, the original SHE erosion and sediment yield component, forms the primary basis for the SHETRAN sediment component (Wicks, 1988; Wicks and Bathurst, in press; Bathurst et al, 1995). The revision of the SHESED model to form the SHETRAN sediment component is largely based on the literature review by Purnama and Bathurst (1991) which covers sediment transport in overland and channel flow.

Symbols are generally defined where they are used and the relevant SI units are indicated. They are also defined in a symbols list at the end of the document, with dimensions indicated.

The process and routing equations are presented in turn for the hillslope and channel subcomponents, and the general approach to solving the routing equations is given as an algorithm.

# 2 HILLSLOPE EROSION AND SEDIMENT TRANSPORT PROCESSES

## 2.1 Processes

The processes simulated are:

1) soil erosion by raindrop impact and leaf drip impact;
2) soil erosion by overland flow;
3) two-dimensional convection of eroded sediment, routed as total load and afterwards divided into different size fractions.

The subcomponent first determines the amount of soil erosion using details of the applied rainfall and resulting overland flow provided by the SHETRAN flow component. Here, erosion refers simply to the detachment of soil particles from the soil mass composing the ground, not to any transport. The eroded material is then routed across the model grid system according to the overland flow calculated by SHETRAN.

## 2.2 Raindrop and Leafdrip Impact Erosion

### 2.2.1 Erosion Equation

The rate of soil erosion by raindrop and leaf drip impact is determined with the following empirical equation, which is derived from one given by Wicks (1988):

$$D_r = k_r F_w (1 - C_g - C_r)(M_r + M_d) \tag{1}$$

Here $D_r$ = rate of detachment of soil ($kg\,m^{-2}s^{-1}$); $k_r$ = raindrop impact soil erodibility coefficient ($J^{-1}$); $C_g$ = proportion of ground shielded by near ground cover (decimal fraction); $C_r$ = proportion of ground shielded by ground level cover (decimal fraction); $M_r$ = momentum squared of raindrops reaching the ground per unit time per unit area ($kg^2s^{-3}$); $M_d$ = momentum squared of leaf drip reaching the ground per unit time per unit area ($kg^2s^{-3}$). Note that the cover fractions $C_g$ and $C_r$ are explained more fully in Section 2.4.

Following Park et al (1982), the protective effect of a surface water layer in reducing the energy imparted to the soil by raindrop and leaf drip impact is accounted for by $F_w$, where

$$F_w = \begin{cases} 1 & \text{if } h \le d_m \\ \exp\left(1 - \frac{h}{d_m}\right) & \text{if } h > d_m \end{cases} \tag{2}$$

Here h = water depth (m); and $d_m$ = effective drip/drop diameter (m) given by

$$d_m = \max\left[d_{min}, d_l\left(\frac{DRAINA}{PNETTO}\right), 0.01935 PNETTO^{0.182}\right] \tag{3}$$

where $d_{min}$ is specified for computational convenience as a minimum value allowed for $d_m$ (chosen as $10^{-4}$m); $d_l$ = leaf drip diameter (m); DRAINA = water drainage rate from canopy obtained from SHETRAN (m $s^{-1}$); and PNETTO = net precipitation rate including drainage from canopy obtained from SHETRAN (m $s^{-1}$). For computational purposes, $d_m$ is set to $d_{min}$ when PNETTO is zero. The last of the three options in the maximum bracket in Equation 3, used to represent the effective raindrop size, is given by Laws and Parsons (1943). The middle term was chosen to describe the increasing dominance of leafdrips over raindrops when the net precipitation striking the ground is dominated by drainage from the vegetative canopy.

If eroded sediment accumulates on the ground to a user specified depth of DLSMAX (m) or greater then it is assumed to protect the soil from further erosion, and $D_r$ is set to zero. This is to prevent the soil erosion continuing unrealistically.

Following the suggestion of Simons et al (1982), detached soil eroded by raindrop and leaf drip impact initially retains the size distribution of the soil forming the ground.

2.2.2 Rainfall Drop Momentum

To calculate the average raindrop momentum it is necessary to know the distribution of raindrop sizes and the raindrop intensity throughout the storm. Following Wicks (1988), the model assumes that the drop size distribution is that given by Marshall and Palmer (1948). Using this assumption, the average raindrop momentum squared per unit time, $M_{av}$($kg^2\ m^2\ s^{-3}$), is

$$M_{av}=A_1 a_1 I^{b_1} \quad (4)$$

where $A_1$ is the area over which $M_{av}$ is averaged ($m^2$); I is the rainfall intensity averaged over $A_1$ ($mm\,h^{-1}$); and $a_1$ and $b_1$ are given in the table below.

| Range for I($mm\,h^{-1}$) | $a_1$ | $b_1$ |
|---|---|---|
| 0 - 10 | $2.6893\times10^{-8}$ | 1.6896 |
| 10 - 50 | $3.7514\times10^{-8}$ | 1.5545 |
| 50 - 100 | $6.1192\times10^{-8}$ | 1.4242 |
| ≥ 100 | $11.737\times10^{-8}$ | 1.2821 |

Over an area of hillslope, $A_1$ , the spatially averaged momentum squared of rainfall reaching the ground, per unit time, per unit area is obtained from Equation 4 as

$$M_r=(1-C_c)a_1 I^{b_1} \quad (5)$$

where $C_c$ is the proportion of ground shielded by the dominant vegetation canopy, and the other symbols are defined above. The factor of ($1-C_c$) is necessary since although $M_r$ is averaged over the area $A_1$ , the area of ground that raindrops strike is only $A_1\times(1-C_c)$. Further discussion of the definition of $C_c$ is given in Section 2.4.

Note that the Marshall and Palmer distribution is not completely general and may in particular not provide a good fit to observed distributions at high rainfall intensities. The model code has been written in such a way as to allow an alternative rainfall drop size distribution to be introduced in the future.

2.2.3 Leaf Drip Momentum

Momentum squared per unit time per unit area for leaf drip onto the ground, $M_d$, is calculated using Equations 6, 7 and 8 below (Wicks, 1988):

$$M_d = \frac{\pi}{6} V_d^2 \rho^2 d_l^3 \times L_d \times DRAINA \tag{6}$$

where g = acceleration due to gravity (m $s^{-2}$), $\rho$ = density of water (kg $m^{-3}$), $L_d$ = proportion of drainage that falls as leaf drip, $d_l$ and DRAINA are defined as before, and $V_d$ is the leaf drip fall speed ($ms^{-1}$) given by

$$V_d = \sqrt{\frac{M}{\beta} g (1 - e^{-\frac{2X\beta}{M}})} \tag{7}$$

Here X = average leaf drip fall distance (m), M = the average mass of leaf drips (kg), $\beta$ = the friction constant (kg $m^{-1}$) and g = acceleration due to gravity (m $s^{-2}$ )

$$\frac{M}{\beta} = a_2 + b_2 d_l \tag{8}$$

where $a_2$ and $b_2$ are given in the following table.

| Range for $d_l$ (m) | Range for X (m) | $a_2$ | $b_2$ |
|---|---|---|---|
| <0.0033 | all X | 0 | 2200 |
| ≥0.0033 | <7.5 | 1.93 | 1640 |
| ≥0.0033 | ≥7.5 | 5.14 | 660 |

## 2.3 Overland Flow Erosion

### 2.3.1 Erosion Equation

The rate of erosion by overland flow, considered as uniform sheet erosion on the hillslopes of a catchment, is determined using the approach of Ariathurai & Arulanandan (1978):

$$D_q = \begin{cases} k_f(1-C_r)\left[\dfrac{\tau}{\tau_{ec}} - 1\right] & \text{if } \tau > \tau_{ec} \\ 0 & \text{otherwise} \end{cases} \qquad \textbf{(9)}$$

Here $D_q$ = the rate of detachment of soil per unit area ($kg\,m^{-2}s^{-1}$); $k_f$ = overland flow soil erodibility coefficient ($kg\,m^{-2}s^{-1}$); $\tau$ = shear stress due to overland flow ($N\,m^{-2}$); $\tau_{ec}$ = critical shear stress for initiation of sediment motion ($N\,m^{-2}$). If loose sediment accumulates on the ground to a user specified depth of DLSMAX or greater then it is assumed to protect the soil from further erosion, and $D_q$ is set to zero. This is to prevent soil erosion continuing unrealistically.

In the first instance detached soil eroded by overland flow retains the size distribution of the soil forming the ground.

### 2.3.2 Shear Stress

The shear stress, $\tau$ is given by

$$\tau = \rho g h S \qquad \textbf{(10)}$$

where $\rho$ = water density ($kg\,m^{-3}$); g = acceleration due to gravity ($m\,s^{-2}$); h = water depth (m); S = water surface slope in the direction of flow ($m\,m^{-1}$).

The critical shear stress, $\tau_{ec}$, may be calculated by using either the Shields (1936) curve or a simple alternative empirical expression. The choice of method is made by the model user. The Shields curve, extended to cover small particle sizes by Mantz (1977) is expressed as:

$$\tau_{ec}=(\rho_s-\rho)gD_{50}a_3R_*^{b_3} \qquad (11)$$

where $\rho_s$ = the density of sediment particles (kg m$^{-3}$); $D_{50}$ is the sediment particle diameter that is greater than the diameter of 50% of the particles (m); $R_*$ is the particle Reynolds number; $a_3$ and $b_3$ are given in the table below; and ρ and g are as defined above.

| Range for $R_*$ | $a_3$ | $b_3$ |
|---|---|---|
| 0.03 - 1 | 0.1 | -0.3 |
| 1 - 6 | 0.1 | -0.62 |
| 6 -30 | 0.033 | 0 |
| 30 - 135 | 0.013 | 0.28 |
| 135 - 400 | 0.03 | 0.1 |
| > 400 | 0.056 | 0 |

$R_*$ is then calculated as

$$R_*=\max\left[0.03,\frac{D_{50}(\tau/\rho)^{0.5}}{\nu}\right] \qquad (12)$$

where ν = water viscosity (m$^2$ s$^{-1}$); and the other symbols are as defined above. The first term provides a lower limit on the Reynolds number otherwise defined by the second term.

SHETRAN provides the following expression for critical shear stress, obtained experimentally by Smerdon & Beasley (1961), as an alternative to the Shields curve which was developed for non-cohesive sediments,

$$\tau_{ec}=0.493\times10^{1.83F_c} \qquad (13)$$

where $F_c$ is the fractional clay content of the sediment by weight. The clay content consists of all the particles smaller than 0.002mm in diameter in this case.

2.3.3 Algorithm

To run the model, the hillslope is divided into grid elements. In calculating the overland flow erosion within one grid element, it was found to be most appropriate to consider only the erosion due to flow in or out of the element across one of its four faces. The face across which the flow into or out of the element is greatest is chosen, allowing the most accurate estimation of the total overland flow erosion in the grid square possible. Note that, once eroded, the sediment discharged from each grid element is routed out across each of the four faces in proportion to water discharge.

So, to calculate the overland flow erosion in a grid element:

1. The maximal flow face is chosen.
2. The slope, S, across this face is calculated. The model calculates S from water depths if flow is between hillslope elements. However if flow is from the hillslope to a channel element, the bank-full water height in the channel is used.
3. The above equations are used to calculate the volume of sediment eroded by the flow in the chosen direction.
4. The erosion due to flow in the chosen direction is assumed to represent the total overland flow erosion in the grid element.

## 2.4 Vegetation, Rock and Canopy Cover

There are three types of cover used by the sediment model. Ground cover ($C_g$) is near-ground cover that shields soil from raindrop and leaf drip impact only, and may consist of low-lying vegetation, vegetation litter and snow. Rock cover ($C_r$) provides protection both from drip/drop erosion and overland flow erosion, and is defined independently of $C_g$. Rock cover may consist of vegetation litter, mulch, stones and rock or concrete surfaces. If ground and rock cover are defined such that $C_g + C_r = 1$, then $D_r$ is zero.

Canopy cover ($C_c$) is the proportion of the ground covered by the dominant vegetation type, directly calculated from the SHETRAN parameters PLAI (plant leaf area index) and CLAI (canopy leaf area index) (Parkin, 1995).

$$C_c = PLAI \times \min[CLAI, 1] \qquad (14)$$

This cover is introduced to provide a facility for modelling vegetation (taller than the ground cover) which shields the soil from the direct impact of raindrops but allows rainwater to coalesce on its surface and fall to the ground as leaf drips, which themselves can cause significant erosion. Where the dominant vegetation is low-lying (less than 0.5m tall), such as grass, this is still described by $C_c$, and $C_g$ refers to any drip-shielding cover other than this.

## 2.5 Capacity for Overland Transport

### 2.5.1 Capacity Equations

The total volumetric transport capacity of sediment particles in the overland flow across the hillslope is calculated by using one of two equations selected by the model user, as recommended by the studies of Julien & Simons (1985), and Park et al (1982).
Either the Yalin equation (Yalin 1963) is used

$$G_{tot} = 0.635 \sqrt{\frac{\tau}{\rho}} l D_{50} \delta [1 - \frac{1}{a\delta} \ln(1 + a\delta)] \qquad (15)$$

or the Engelund-Hansen equation (Engelund & Hansen, 1967):

$$G_{tot} = \begin{cases} \dfrac{0.05 Q^2 S^{\frac{3}{2}}}{\sqrt{gh}\left(\dfrac{\rho_s}{\rho}-1\right)^2 D_{50} l} & \textit{if } h>0 \\ =0 & \textit{otherwise} \end{cases} \tag{16}$$

Here $G_{tot}$ is the capacity particulate transport rate for overland flow (including all sediment size groups) ($m^3 s^{-1}$); l = width of the flow; Q = the water flow rate ($m^3 s^{-1}$); δ and a are defined below; and the rest of the symbols are defined as in Section 2.3. Note that $G_{tot}$ represents volume of sediment particles so the mass flux is given by $G_{tot}$ multiplied by particulate density.

$$\delta = \max[0, \frac{\tau}{\tau_{ec}} - 1] \tag{17}$$

$$a = 2.45 \sqrt{\frac{\tau_{ec}}{[(\rho_s - \rho) g D_{50}]}} \cdot \left(\frac{\rho_s}{\rho}\right)^{-0.4} \tag{18}$$

Here all the symbols are defined as in Section 2.3, although it should be noted that $\tau_{ec}$ is calculated from the Shields formula, and cannot be determined from Equation 13.

Equations 15 and 16 were developed for sediment transport in channels, and their applicability to overland flow has not been fully explored. In addition, the equations were produced primarily for noncohesive sediments and their applicability to cohesive sediments has not been satisfactorily examined. In the light of this, the total sediment flow capacity is not allowed to be any greater than a maximum value set by the user, thus ensuring that inappropriate predictions of Equations 15 and 16 do not result in impractically high simulated sediment transport. The total particulate sediment flow capacity, G ($m^3 s^{-1}$), is therefore given by

$$G = \min[G_{tot}, Q \times FPCRIT] \tag{19}$$

where $G_{tot}$ and Q are defined above, and FPCRIT is maximum sustainable sediment concentration specified by the user. FPCRIT is non-dimensional, and is the ratio of the particle volume to water volume. (Currently one value of FPCRIT applies for both overland and channel flow).

2.5.2 Algorithm

The hillslope is divided into grid elements. The sediment flow out of an element is determined by considering flow across each of the outflow faces of the element in turn (a maximum of four faces).

1. The above equations are used once for each outflow face, to determine the capacity of the flow to carry sediment across each of the outflow faces.
2. The sum of these capacities is the total particulate sediment flow capacity.
3. This total capacity is limited by FPCRIT multiplied by the total water outflow from the grid element.

## 2.6 Routing Overland Sediment Transport

2.6.1 Routing Equation

The following two-dimensional mass conservation equation is applied to each sediment size fraction in turn:

$$\frac{\partial(c_i h)}{\partial t}+(1-\lambda)\frac{\partial z_i}{\partial t}+\frac{\partial g_{xi}}{\partial x}+\frac{\partial g_{yi}}{\partial y}=0 \qquad \textbf{(20)}$$

where h = water depth (m); c = sediment concentration ($m^3\,m^{-3}$); $\lambda$ = loose sediment porosity (decimal fraction); z = depth of loose soil (m); $g_x$ and $g_y$ = volumetric sediment transport rates per unit width in the x and y directions respectively ($m^3\,s^{-1}\,m^{-1}$); t = time (s); and i = size fraction.

Following Bennett (1974), the dispersion effect has been neglected in comparison with the effects of other processes embodied in Equation 20.

The actual rate of transport of sediment is limited by the carrying capacity (see Section 2.4), and the speed of sediment particles moving in the flow is assumed to equal the speed of the water flow. The portion of the sediment that cannot be carried by the flow is left on the hillslope as loose sediment.

2.6.2 Routing Algorithm

The hillslope is divided into grid elements, and the sediment carried out of one element during one time step is determined in the following manner.

1. The sediment is assumed to be incompressible, which means that the mass balance equation is also a volume balance equation. Using this assumption, Equation 20 is discretised and used to find the volume of sediment in each separate size group that is available for transport during the time step, $VDSED_i$. This includes the sediment (in motion and deposited as loose sediment on the ground) that was in the element at the beginning of the time step, plus the sediment eroded within the element and that carried in from other elements during the time step. The total volume of available sediment in all the sediment size groups is $\Sigma VDSED_i$.

   Knowing the total water available to flow out of the element in the time step, and the water discharge rate from the element, the potential sediment outflow rate based on availability, $G_{sum}$ ($m^3 s^{-1}$) is then calculated.

2. The total available volume of sediment in all the size groups is compared with the maximum sediment volume, G, that can be transported out of the grid element during the time step, (calculated from Equation 19). The actual total sediment volume flowing out of the element during the time step is equal to either the volume capacity of the flow, or the sediment volume available during the time step, whichever is the smallest. Excess sediment that cannot be carried by the flow remains in the element.

In other words:

If $G > G_{sum}$ then the overland flow is capable of carrying all of the available sediment, and the total outflow of sediment equals the potential $G_{sum}$.

If $G \leq G_{sum}$ then the overland flow is saturated, and the total outflow of sediment equals the capacity outflow, G.

3. The sediment that flows out of the grid element, and that remaining there at the end of the time step, is divided into size fractions in the same ratio as the available sediment. The sediment carried out of the element across all the outflow faces is divided in proportion to the water flow rate across the different outflow faces. (Although sediment discharge may not vary linearly with water discharge, errors arising from adopting these proportions are likely to be small compared with other uncertainties).

4. The concentration of mobile sediment remaining in the grid element at the end of the time step is the same as the concentration in the flow transported out of the element. Any additional sediment not carried by the flow settles to form loose sediment. Thus steps 2 and 4 determine whether there is net deposition or erosion of loose sediment in the element.

## 3 Channel Erosion and Sediment Transport Processes

### 3.1 Processes

The processes simulated are

1) Channel bank erosion by channel flow;
2) Deposition and mobilisation of channel bed sediment;
3) One-Dimensional convection of sediment, routed by size fraction in channel flow;
4) Infiltration of fine sediment into the channel bed;
5) Armouring of channel bed;
6) Input of sediment with overland flow.

Details of the possible amounts of erosion, deposition and other exchanges are determined first. The sediment load is then routed along the channel system. Sediment is divided into different size groups. There is a maximum of one "fine" sediment size group (representing all the sediment, cohesive and non-cohesive, that has a diameter of <0.25mm). There are a maximum of 6 "non-fine" sediment size groups, representing larger sized sediment. Although change in bed elevation is implied by erosion and deposition, such change is small enough to be neglected as far as the flow calculations are concerned. There is no feedback to the flow component concerning either bed elements or sediment concentrations.

### 3.2 Channel Bed Erosion

Erosion of the bed by the flow is not simulated with an explicit equation (as for the overland flow erosion). An explicit representation of soil erosion is required for overland flow erosion because soils are generally cohesive and soil particles have first to be detached from the soil mass before they become available as loose material for transport. However, in river channels the bed material is generally non-cohesive so the sediment particles are already loose. The only controls on bed erosion are then the thickness of the available part of the bed material, the ability of the flow to move the range of available particle sizes, and the transport capacity of the flow.

The channel bed is divided into two layers, using the same principle as Park & Jain (1987). The top ("active") layer is available for mobilisation by channel water flow. The bottom layer is out of reach of the water flow. The total thickness of the top bed sediment layer (all the sediment size groups), $d_c$, is given by

$$d_c = \begin{cases} \delta_0 & \textit{if } DLS > \delta_0 \\ DLS & \textit{otherwise} \end{cases} \qquad (21)$$

where $\delta_0$ = maximum depth of the top bed sediment layer (m); and DLS = total depth of loose bed sediment (m) (time-varying and calculated by the model). $\delta_0$ may be set by the user, although typically for gravel-bed rivers it can be set equal to the initial $D_{99}$ size of the bed material, where $D_{99}$ is the sediment diameter for which 99% of the sediment is finer (Wicks, 1988).

The transfer of sediment between bed layers, and the change in overall bed surface elevation, reflecting erosion or deposition, is an outcome of the solution of the routing equation. The routing equation also incorporates a bed armour and an infiltration component. The algorithm for computing the changes in bed sediment is given in Section 3.8.2.

## 3.3 Channel Bank Erosion

### 3.3.1 Equations

Following Osman & Thorne (1988), the rate of erosion by channel flow at one of the two channel banks is determined using

$$E_b = \begin{cases} k_b \left[ \dfrac{\tau_b}{\tau_{bc}} - 1 \right] & \textit{if } \tau_b > \tau_{bc} \\ 0 & \textit{otherwise} \end{cases} \qquad (22)$$

where $E_b$ = rate of detachment of material per unit area of bank ($kg\,m^{-2}\,s^{-1}$); $k_b$ = bank erodibility coefficient ($kg m^{-2} s^{-1}$); $\tau_{bc}$ is the critical shear stress for initiation of motion of bank material ($N m^{-2}$); and $\tau_b$ is the shear stress acting on the bank ($N m^{-2}$). The total bank erosion rate is two times $E_b$ since there are two channel banks. Based on the work of Osman and Thorne (1988), the shear stress at the channel bank can be related to the shear stress on the channel bed in the following manner.

$$\tau_b = K\tau \tag{23}$$

where $\tau$ is the mean flow shear stress on the bed

$$\tau = \rho g H S \tag{24}$$

and K is a proportionality constant calculated (assuming a rectangular channel), as

$$K = a_4 + b_4 \frac{B}{H} \tag{25}$$

Here $a_4$ and $b_4$ are given in the following table

| Range for B/H | $a_4$ | $b_4$ |
|---|---|---|
| < 1 | 0.05 | 0.41 |
| 1 - 2 | 0.24 | 0.22 |
| 2 - 4 | 0.61 | 0.035 |
| ≥ 4 | 0.75 | 0.0 |

B = effective channel width (m); H = flow depth (m); S = longstream water surface slope ($m m^{-1}$); $\rho$ = water density ($kg m^{-3}$); and g = acceleration due to gravity ($m s^{-2}$).

The critical shear stress at the bank, $\tau_{bc}$, is obtained in the same way as the critical shear stress for overland flow (see Section 2.3.2). Either the Shields curve, or the alternative formula based on the clay fraction of the soil in the bank, can be used.

The above equation strictly covers bank erosion due to flow shear stress only. Geotechnical failure must be accounted for empirically with the erodibility coefficient.

In the first instance, it is assumed that detached material retains the size distribution of the bank material.

### 3.3.2 Algorithms

The channel is divided into link elements. As with overland flow erosion, a maximum outflow face is selected. The bank erosion rate within a link element is taken to be that due to flow out of the element across its maximum outflow face. Any additional flow in other directions does not contribute to bank erosion. Note that, once eroded, any sediment discharged from a channel element is routed in every outflow direction in proportion to water discharge.

## 3.4 Capacity for Sediment Transport

The maximum or capacity sediment transport rate of the channel flow is required for the routing algorithm. The capacity rate for carrying sediment is determined for each separate sediment size group.

Fine material (representative diameter < 0.25mm) is assumed to travel at the speed of the water flow (Wicks, 1988). Water flow can usually transport all the available fine sediment, so sediment transport capacity could be said to be infinite (Wicks, 1988). However, since the sediment routing algorithms used by SHETRAN are not intended to be able to represent mud flow, a maximum sustainable particulate concentration of fines in channel flow is specified by the user, using the parameter FPCRIT. The capacity transport rate of fine material then follows from the speed and capacity concentration of fines. (Currently one value of FPCRIT applies for both overland and channel flow).

The other sediment size groups (representative diameters > 0.25 mm) are called "non-fine" sediment size groups here. Following the practice of Wicks (1988), the capacity sediment transport rate in each non-fine sediment size group is given by one of the three transport equations shown below.

1) The Engelund-Hansen total load equation (Engelund & Hansen, 1967)

$$G_i = \frac{0.05 B\, U^2 H^{1.5} S^{1.5}}{(s-1)^2 D_i\, g^{0.5}} \tag{26}$$

where $G_i$ = volumetric sediment transport rate for particles in size group i ($m^3 s^{-1}$); B = flow width (taken as bank-full width of channel) (m); U = water velocity ($m\,s^{-1}$); H = flow depth (m); S = water surface slope ($m\,m^{-1}$); s = sediment specific gravity (decimal fraction); $D_i$ = representative sediment particle diameter for the size group (m); and g = acceleration due to gravity ($m\,s^{-2}$). The suggested applicability of the Engelund-Hansen equation is for $(D_{75}/D_{25})^{0.5} < 1.6$ and for a particle mean fall diameter greater than 0.15 mm.

2) The Ackers-White total load equation (Ackers & White, 1973)

$$G_i = Q \frac{D_i}{H} \left( \frac{U}{u_*} \right)^{n_i} G_{gr,i} \tag{27}$$

where $G_i$ = volumetric sediment particle transport rate for particles in size group i ($m^3 s^{-1}$); Q = water discharge ($m^3 s^{-1}$); $D_i$ = representative sediment particle diameter for the sediment size group i (m); H = water flow depth (m); U = mean water flow velocity ($m\,s^{-1}$); $u_*$ = shear velocity ($m\,s^{-1}$); $n_i$ = the transition exponent for sediment size group i; and $G_{gr,i}$ = dimensionless sediment transport rate for sediment size group i.

The shear velocity u* is given by

$$u_* = \sqrt{gHS} \tag{28}$$

where g = acceleration due to gravity (m $s^{-2}$); H = as above; and S = water surface slope (m $m^{-1}$).

The procedure for applying the Ackers-White equation is as follows (Ackers & White, 1973 ):

(i) Determine the dimensionless sediment diameter, $D_{gr,i}$

$$D_{gr,i} = D_i \left[ \frac{g(s-1)}{\nu^2} \right]^{\frac{1}{3}} \tag{29}$$

where s = sediment specific gravity (decimal fraction); ν = kinematic viscosity of water ($m^2$ $s^{-1}$); and $D_i$ and g are defined above.

(ii) Determine the transition exponent, $n_i$, the initial motion parameter, $A_i'$, the coefficient $k_i$ and exponent, $m_i$, in the sediment transport function. These are determined as shown in the following tables.

| Range for $D_{gri}$ | fine<br><1 | coarse<br>>60 |
|---|---|---|
| $n_i$ | 1 | 0 |
| $A_i'$ | 0.37 | 0.17 |
| $m_i$ | 11 | 1.5 |
| $k_i$ | $2.95 \times 10^{-4}$ | 0.025 |

| Range for $D_{gr,i}$ | intermediate<br>1-60 |
|---|---|
| $n_i$ | $1 - 0.56 \log D_{gr,i}$ |
| $A_i'$ | $(0.23/\sqrt{D_{gr,i}}) + 0.14$ |
| $m_i$ | $(9.66/D_{gr,i}) + 1.34$ |
| $\log k_i$ | $2.86 \log D_{gr,i} - (\log D_{gr,i})^2 - 3.53$ |

(iii) Determine the particle mobility, $F_{gr,i}$

$$F_{gr,i} = \frac{u_*^{n_i}}{[gD_i(s-1)]^{0.5}} \times \left[ \frac{U}{5.657 \log_{10}(\frac{10H}{D_i})} \right]^{(1-n_i)} \tag{30}$$

(iv) Determine the dimensionless sediment transport rate, $G_{gr,i}$.

$$G_{gr,i} = \begin{cases} k_i \left[ \frac{F_{gr,i}}{A_i'} - 1 \right]^{m_i} & \textit{if } A_i' < F_{gr,i} \\ 0 & \textit{otherwise} \end{cases} \tag{31}$$

(v) Determine the sediment discharge with Equation 27.

The suggested applicability for this equation is for $D_{gr,i} \geq 1$ and for flows with Froude numbers less than 0.8.

3) The Day modification to the Ackers-White equation (Day, 1980). This seeks to account for the effects of particle hiding and exposure on the initiation of motion of nonuniform bed material. It is relevant to gravel-bed channels. The procedure is as follows:

(i) Determine the $D_{16}$, $D_{50}$ and $D_{84}$ sizes of the bed material in the active bed layer of the channel. These are the particle diameters for which 16%, 50% and 84% of the particles are smaller.

(ii) Determine the critical diameter $D_a$ which is the particle size in a nonuniform bed material which would begin to move at the same flow conditions as would a uniform bed material of size $D_a$ (ie which is unaffected by hiding and exposure effects)

$$D_a = 1.62\ D_{50}\left(\frac{D_{84}}{D_{16}}\right)^{-0.28} \quad (32)$$

(iii) Determine the dimensionless sediment diameter for the critical diameter, $D_{gra}$, using Equation 29.

$$D_{gra} = D_a\left[\frac{g(s-1)}{\nu^2}\right]^{\frac{1}{3}} \quad (33)$$

(iv) Determine the initial motion parameter for $D_a$

$$A' = \begin{cases} \dfrac{0.23}{\sqrt{D_{gra}}} + 0.14 & \textit{if } 60 \geq D_{gra} \geq 1 \\ 0.17 & \textit{otherwise} \end{cases} \quad (34)$$

The remaining steps in the procedure are repeated for all size fractions.

(v) Determine the initial motion parameter for size fraction i.

$$A_i' = A'\left[0.4\left(\frac{D_i}{D_a}\right)^{-0.5} + 0.6\right] \quad (35)$$

(vi) Determine $n_i$, $m_i$ and $k_i$ for each size fraction i, using $D_{gr,i}$ and the tables shown for the standard Ackers-White formula application above.

(vii) Determine $F_{gr,i}$ using Equation 30.

(viii) Determine $G_{gr,i}$ using Equation 31.

(ix) Determine $G_i$ from Equation 27.

The Day modification is not recommended for $(D_{84}/D_{16})^{0.5}$ less than about 1.4, since, from Equation 32, $D_a$ does not tend towards $D_{50}$ as it should do as $(D_{84}/D_{16})$ tends to unity. It is also not recommended for $(D_{84}/D_{16})^{0.5}$ greater than about five.

One of the above three transport equations is used to determine the transport capacity, $G_i$, for the sediment particles in each separate non-fine size group. However, the transport equations shown here were derived for sediment composed of a uniform particle diameter. So, when they are used to find the transport capacity for an individual sediment size group, the result they give is as if that size group constituted the entire sediment load. There is therefore a need to find an effective sediment transport capacity for each non-fine sediment size group, $G_{eff,i}$, taking into account the contributions of other sediment size groups. Following Wicks (1988), $G_{eff,i}$ is given by

$$G_{eff,i} = f_i G_i \tag{36}$$

where $f_i$ is the proportion of the available non-fine sediment in size group i, and available sediment refers to the sediment that could potentially be carried by the channel flow. The available sediment is represented by that being carried by the flow and that present in the active layer of the bed. So the fraction $f_i$ is taken to be

$$f_i = \max\left[0.05, \frac{FDEL_i}{\Sigma FDEL_i}, \frac{d_{c,i}}{d_c}\right] \tag{37}$$

where $FDEL_i$ = volumetric concentration of mobile sediment carried by the water in the non-fine sediment size group i; the sum of $FDEL_i$ is taken over all the non-fine sediment size groups; $d_{c,i}$ = notional depth of sediment in the top bed sediment layer in size group i; and

$d_c$ = notional total depth of non-fine sediment in the top bed sediment layer. (For computational convenience, each sediment size is considered to occupy a certain notional depth or thickness. The summation of these depths over all the size groups equals the total bed sediment thickness.)

The user specifies a maximum value for the concentration of sediment sustainable in the channel flow for each separate sediment size group, FPCRIT. Since the sediment particle speed is also known (Equations 46 and 47), FPCRIT provides a cap on the effective sediment transport rate, $G_{eff,i}$. So, aside from its function to limit the total sediment concentration in overland flow, the single value, FPCRIT provides a limit on the concentration of each separate sediment size group in channel flow (the only limit in the case of fine material).

### 3.5 Fine Sediment Deposition and Re-suspension

For the fine sediment particles normally transported in suspension, deposition from the flow can occur at shear stresses below some value critical for deposition, $\tau_{dc,f}$. The rate of deposition is determined using the approach of Ariathurai & Krone (1976):

$$E_{df} = \begin{cases} B W_f c_f \rho_s \left[1 - \dfrac{\tau}{\tau_{dcf}}\right] & \text{if } \tau < \tau_{dcf} \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

where the fine sediment settling velocity $W_f$ ($ms^{-1}$) is calculated from

$$W_f = \frac{D_f^2 g}{18\nu}\left[\frac{\rho_s}{\rho} - 1\right] \tag{39}$$

where $E_{df}$ = rate of deposition of fine sediment per unit length of channel ($kg\,s^{-1}\,m^{-1}$); B = effective width of channel (m); $c_f$ = concentration of fine particles in suspension ($m^3 m^{-3}$); $D_f$ = fine sediment particle size (m); g = acceleration due to gravity ($m\,s^{-2}$); $\nu$ = kinematic viscosity of water ($m^2 s^{-1}$); $\rho_s$ = sediment particle density ($kg\,m^{-3}$); $\rho$ = water density ($kg\,m^{-3}$).

There is currently no available equation for calculating $\tau_{dc,f}$, and its evaluation must therefore be a matter of empirical calibration. This is achieved in the model by relating $\tau_{dc,f}$ to $\tau_{ec,f}$, the critical shear stress for re-suspension of fine material.

$$\tau_{dc,f} = \alpha \, \tau_{ec,f} \tag{40}$$

where $\alpha$ is a constant (less than or equal to 1) specified by the user; and $\tau_{ec,f}$ is obtained in the same way as the critical shear stress for overland flow $\tau_{ec}$ (Equation 11), with $D_f$ replacing $D_{50}$.

All fine material in the top bed sediment layer that is not subject to infiltration and is not held in the layer (as described below) is mobilised by the flow, subject to the limit on mobile sediment concentration provided by FPCRIT. Infiltrating fine sediment is transported to the bottom bed sediment layer. Fine sediment held in the top bed sediment layer plus any sediment that can not be carried by the flow remains in the top bed sediment layer.

### 3.6 Infiltration of Fine Sediment

A single-stage infiltration process is simulated, involving the penetration of fine sediment into the bottom bed sediment layer. The work of Einstein (1968) suggested that the rate of infiltration of fine sediment into the channel bed is limited by the rate at which fine particles settle, and is proportional to the concentration of fine material suspended in the flow. The following formula for infiltration rate, based on Einstein's work, also depends on a user-defined critical particulate concentration of suspended fine sediment, below which no infiltration can take place, FICRIT (given as a ratio of the fine sediment particle volume to water volume).

$$\frac{dc_{df}}{dt} = \begin{cases} \dfrac{W_f}{d_d}(c_f - FICRIT) & \text{if } c_f > FICRIT \\ 0 & \text{otherwise} \end{cases} \tag{41}$$

where $c_{df}$ = concentration of fine sediment in the bottom layer (ratio of volume of fine sediment particles to total bulk volume of sediment in the bottom layer); $W_f$ = average fine sediment particle settling velocity ($m\,s^{-1}$); $d_d$ = depth of the bottom bed sediment layer (m); $c_f$ = concentration of fine sediment mobilised by the flow. The driving mechanism is based on the difference between the concentration of fine sediment in suspension in the flow and a user defined critical concentration in the flow, FICRIT.

It is widely accepted that infiltration will cease if the pores in a channel bed become clogged by fine sediment (e.g. Frostick et al, 1984). Clogging is represented in SHETRAN by a simple cut-off fraction of fine sediment in the bed, above which there can be no infiltration. In this way, infiltration can take place only if the following condition is met

$$FBETA_f < FBICRIT \qquad \textbf{(42)}$$

where $FBETA_i$ = fraction of bed sediment (in both top and bottom layers) that is in the fine sediment size group; and FBICRIT is a user specified critical value of fine sediment concentration in the bed.

The rate of infiltration of fine sediment into the bottom bed sediment layer is limited by the rate at which fine sediment settles to the channel bed (determined by Equation 39). This rate is in turn limited by the fine sediment available in the channel from inflow and erosion.

### 3.7 Mobilisation of Channel Bed Sediment

Using the concept from Section 3.2 of an "active" bed sediment layer (e.g. Bennett and Nordin, 1977; Park & Jain, 1987), only sediment from the top channel bed sediment layer is assumed to be available for mobilisation by channel flow. This concept allows the process of armouring to be simulated, whereby the smaller sizes of material present in the channel bed are protected from being mobilised by the flow by the presence of larger material. Effectively, armouring is an important process if the bed contains gravel and larger material but not if the bed material is only sand or finer (Park & Jain, 1987).

### 3.7.1 Armouring of the Channel Bed

Armouring is modelled by a process of selective entrainment of the finer sizes of a bed sediment with a non-uniform size distribution The process occurs at flow discharges of a strength sufficient to carry finer sizes but not the larger sizes. Consider a bed initially composed homogeneously of a material of a given nonuniform size distribution, the parent material. The finer particle sizes are preferentially removed from the active top bed layer because the flow is incapable of carrying the larger sizes. An equivalent thickness of sediment having the same size distribution as the parent material then passes from the bottom bed layer to the top, replacing the removed material (see Section 3.8.2c). As the process continues, the proportion of the surface layer consisting of the coarser sizes too large to be carried by the flow increases. Sediment transport then decreases. Eventually sediment transport ceases, because the top bed sediment layer is composed of sizes too coarse to be moved by the flow. The bed is then armoured and the top layer is the armour layer. Note that this process depends on the size of the particles comprising the parent sediment. Since the armour layer is composed of material derived from the parent material, the maximum particle size in the armour or surface bed sediment layer cannot exceed the maximum size in the parent material.

The armour layer protects material in the bottom bed sediment layer from further erosion until a flow capable of moving larger sizes, or the full range of sizes, occurs. If the full range is moved, the armour layer is destroyed and must be reformed as the discharge falls. While there is no armour layer, all particle sizes are available for transport (in proportion to their occurrence in the bed material). Fine sediment particles that have infiltrated into the bottom bed sediment layer will also be released at this stage.

### 3.7.2 Fine Material Suspended from Top Bed Sediment Layer

If the shear stress in the channel is greater than the critical shear stress for the suspension of fine material, $\tau_{cc,f}$, then all of the fine material in the top bed sediment layer is assumed to be suspended. However, if the shear stress exerted on the bed is less than the critical value for fine material, then SHETRAN assumes that none of the fine material can be mobilised by the flow. Rather than have a discontinuous jump between full suspension of fines and zero suspension of fines when shear stress is critical, the fraction of fine material in the top bed sediment layer available for mobilisation when the shear stress is below critical is related to the rate at which non-fine sediment is mobilised from the bed. So, during some period of time from time=n to time=n+δt, if the shear stress exerted on the bed by the flow is less than the critical value for fine material, then there is a fraction, $f_{dc}$, of the fine material in the top bed sediment layer that is protected from mobilisation.

$$f_{dc} = \begin{cases} \min\left[1, \dfrac{\Sigma d'_{c,i}}{\Sigma d_{c,i}}\right] & \textit{if } \Sigma d_{c,i} > 0 \\ 0 & \textit{otherwise} \end{cases} \qquad \textbf{(43)}$$

where $d_{c,i}$ = depth of sediment in size group i in the top bed sediment layer at the beginning of the time period; $d'_{c,i}$ = depth of sediment at the end of the time period, following any movement of sediment between the channel bed and the flow; and the sums are taken over all the non-fine sediment size groups.

## 3.8 Channel Sediment Transport

### 3.8.1 Routing Equations

For channel flow transport, mobile sediment is divided into sediment size groups (Simons et al, 1982). The fine sediment size group, (particle diameter < 0.25mm) comprising the cohesive sediment and the fine tail of the non-cohesive sediment, is routed as cohesive sediment. The larger sediment size groups (non-fines) are routed (individually) as non-cohesive sediment.

Dispersion of both fine and non-fine sediment is assumed to be insignificant and is not modelled. No separate allowance for bed and suspended load is made in either case. So, the transport of both fine and non-fine sediment is simulated with the one-dimensional equation for conservation of sediment mass (e.g. Bennett, 1974)

$$\frac{\partial(Ac_i)}{\partial t} + (1-\phi)B\frac{\partial z_i}{\partial t} + \frac{\partial G_i}{\partial x} = q_{si} \tag{44}$$

where A = flow cross sectional area ($m^2$); $c_i$ = concentration of sediment particles in size group i ($m^3 m^{-3}$); $\phi$ = bed sediment porosity; B = active bed width for which there is sediment transport (m); z = depth of bed sediment (m); $G_i$ = volumetric sediment transport rate for the sediment size fraction i ($m^3 s^{-1}$); and $q_{si}$ represents sediment input from bank erosion and overland flow supply per unit channel length for size fraction i ($m^3 s^{-1} m^{-1}$).

There are only three differences between the routing of fine sediment and non-fine sediment in channel flows. The first is the capacity sediment transport rate, $G_i$, the second is the difference in the way that the speed of sediment carried by water flow may be calculated, and the third is the rate at which bed sediment depth changes, $\partial z_i/\partial t$. Fine sediment may be explicitly held in the top bed sediment layer (Section 3.7.2) or may infiltrate to the bottom layer (Section 3.6), thus directly affecting the composition of channel bed sediment. In contrast, these processes are not modelled for non-fine sediment.

Following Wicks (1988), the volumetric sediment transport rate is related to sediment concentration by

$$G_i = c_i U_{si} A \tag{45}$$

where $U_{si}$ = speed of sediment particles in size group i ($m s^{-1}$), and the other symbols are defined above.

The speed of fine sediment particles in the flow, $U_{sf}$ is assumed to be the same as the water flow speed, U (Wicks, 1988).

$$U_{sf} = U \tag{46}$$

The speed of each non-fine sediment size group, $U_{si}$, can be set to equal water flow speed also, or may be determined following Phillips and Sutherland (1985)

$$U_{si} = 8.5 u_* \left[1 - \frac{u_{*ci}}{u_*}\right]^{0.5} \qquad (47)$$

where u* is the bed shear speed ($ms^{-1}$), given as

$$u_* = \left[\frac{\tau}{\rho}\right]^{0.5} = (gHS)^{0.5} \qquad (48)$$

and $u_{*ci}$ is the critical value for initiation of motion for a sediment particle size, i, given as

$$u_{*ci} = \left[\frac{\tau_{eci}}{\rho}\right]^{0.5} \qquad (49)$$

and the critical shear stress, $\tau_{eci}$ is determined from the Shields curve as $\tau_{ec}$ is in Section 2.3.2 with $D_i$ replacing $D_{50}$. In the above, g = acceleration due to gravity ($ms^{-2}$); H = flow depth (m); S = longstream water surface slope ($mm^{-1}$); and ρ = water density ($kgm^{-3}$). Equation 47 is based on an empirical analysis using limited data. Its accuracy cannot therefore be relied on for conditions differing from those used in its derivation, but it is expected to yield more satisfactory results then the assumption that sediment flow speed equals water speed. The value of $U_{si}$ obtained from Equation 47 is constrained to be equal to or less than the water flow speed, U.

### 3.8.2 Algorithm for Sediment Routing

The channel is divided into channel link elements. The calculations start at the upstream end of the river system and proceed in the downward direction. This means that the volume of sediment carried into a channel link during a time step is known, and therefore the volume of sediment carried out of that link during the time step can be determined.

First, non-fine sediment is routed. This has to happen before the fine sediment is routed, because the volume of non-fine sediment present in the top bed sediment layer is used to determine the volume of fine material that is held in the top layer (see 3.7.2).

a) Non-fine Sediment Carried by the Flow

This procedure is followed for each sediment size group in turn.

1. The mass balance equation (Equation 44), is used to find the volume of sediment in the size group that is available for transport during the time step. This includes sediment already in the link (carried by the flow and in the top bed layer), plus the sediment eroded from the banks during the time step, plus sediment carried in from other elements (channel and grid) during the time step.

   The water available in the link during the time step is already known from the flow calculations. So, by assuming that all the available sediment can be carried by the flow (unlimited transport capacity), the potential sediment concentration for the size group is calculated.

2. The capacity sediment concentration is calculated using the capacity transport rate (Section 3.4; Equations 26 or 27), the particle speed (equal to water flow speed or given by Equation 47), and Equation 45 above. The capacity sediment concentration is then compared with the potential sediment concentration (step 1), and the actual concentration of sediment carried by the flow in each size group is set to be the smallest of the two.

   Any sediment in each size group that cannot be carried by the flow, is added to the top bed sediment layer in the link element causing deposition and resulting in sediment transfer from the top to the bottom layer. Where the capacity concentration exceeds that needed to carry the available sediment, there is excess carrying capacity and bed erosion can occur, resulting in transfer of sediment from the bottom layer to the top layer. Whether there is net erosion or deposition of bed sediment in each size group thus depends on the carrying capacity of the flow.

3. The actual volume of each sediment size group transported out of the link depends on the transport rate, which equals the sediment particle speed multiplied by the concentration of sediment carried by the flow, multiplied by the cross-sectional area of flow (Equation 45).

b) Fine Sediment Carried by the Flow

The model routes fine sediment, in the same way as non-fine sediment, as total load (i.e. not distinguishing suspended load from bed load). However, it should be remembered that in reality fine sediment carried by the flow will be entirely suspended.

1. The mass balance equation, (Equation 44) is used to calculate the volume of fine sediment available in the channel link during the time step. As before, this includes sediment already in the link (carried by the flow and deposited in the top bed layer), plus eroded bank sediment, plus sediment carried into the link during the time step (from grid and channel elements).

   The water available in the link during the time step is already known. So, by assuming that all the available sediment can be carried in suspension (unlimited transport capacity), the potential fine sediment concentration is calculated.

2. The volume of fines infiltrating into the bed sediment during the time step is calculated from Equation 41. Accounting for this, the potential concentration of fine sediment in the flow is reduced.

3. The volume of fines held in the top layer is calculated (Equation 43). Accounting for this, the potential concentration of fine sediment in the flow is reduced again.

4. The fine sediment capacity concentration, set by the user to FPCRIT (see Section 3.4) is compared with the potential sediment concentration (step 1). The actual concentration of fine sediment carried by the flow is set to be the smallest of the two.

   Any fine sediment that cannot be carried by the flow, is added to the top bed sediment layer in the link. Whether there is net erosion or deposition of fine sediment thus depends on the carrying capacity, as well as on the volume of infiltrating and armoured fine material.

5. The actual sediment transport rate for fines equals the sediment particle speed (equal to water speed) multiplied by the actual sediment concentration in the flow, multiplied by flow cross-sectional area (Equation 45).

c) Bed Sediment

The model makes the assumption that the bulk density of bed sediment is constant. Thus, in the model's terms, the total depth of sediment in the bed is the sum of the depths in each separate sediment size group. However, the model represents a situation where the bulk bed sediment density actually changes. For example, the density of the bed will change as fine sediment particles infiltrate into the bed, because they fill up pores in between the larger particles. In order to account for this conflict between the model assumptions and the physical environment, the depth of sediment in any given size group is said to represent a notional quantitiy proportional to the mass of sediment in that size group. In this way, changes of density in the bed sediment are represented by a change in bed sediment depth.

1. The depth of each non-fine sediment size group in the top bed sediment layer is augmented or depleted during the time step depending on what the carrying capacity of the flow is. Any fine sediment in the top layer at the end of the time step is composed of material held in the layer plus the fine sediment that could not be carried by the flow. The total depth of sediment in the top layer is the sum of the sediment depths in each size group.

2. The depth of fine sediment in the bottom layer is increased by infiltration (see Section 3.5). The total depth of sediment in the bottom layer is therefore increased.

3. The total depth of sediment in the top bed sediment layer cannot be greater than the maximum, $\delta_0$ (see Section 3.2). So, if the total depth of sediment in the top layer (step 1) is greater than $\delta_0$ then the extra sediment is transferred to the bottom sediment layer.

4. Unless the total bed sediment depth is less than $\delta_0$, the top bed sediment layer is always $\delta_0$ deep (see Section 3.2). So, if the total depth of sediment in the top layer (step 1) is calculated to be less than $\delta_0$, sediment in the bottom layer is transferred to the top layer (until the top layer depth reaches $\delta_0$).

# 4 MODEL PARAMETER AND VARIABLE EVALUATION

The parameters and variables which feature in the erosion and sediment transport calculations are determined by three means: transfer from the SHETRAN hydrology components, provision in a data file by the user and calculation within the sediment component.

## 4.1 Transfers from SHETRAN Hydrology Component

The following information is transferred either directly or with minor processing:

Rainfall: rainfall intensity I, likely to be required at intervals of 10 minutes or less for accurate erosion calculations;

Catchment Morphology: ground and channel bed elevations, channel geometry, channel network, soil porosity;

Flow Variables: overland flow depth, discharge and slope; channel flow cross-sectional area, width, depth, discharge, slope and velocity; canopy drainage.

## 4.2 User-provided Data

The user manual for the sediment code provides a full list of the variables that the user must specify in the sediment input data file, together with the format. The following is a summary and includes likely ranges of parameter values where appropriate. Other important parameters for the sediment model, such as sediment particle density (2650 kg $m^{-3}$), water density, water viscosity, and acceleration due to gravity, are not supplied by the user but are hard-coded in the model.

General parameters

- a single set of representative sediment and soil particle diameters, $D_i$, i=1,2..7;

Silt, clay and fine sand (D <0.25 mm) can form one size fraction; coarser sand and gravel can be divided into size fractions at the discretion of the user. Note: only the smallest sediment size fraction will be taken by the model to be "fine" sediment and treated appropriately.

- flags to determine which capacity sediment transport equations are used
- upper limit on volumetric suspended sediment concentration - this could be set to 1.0 indicating that infinite carrying capacity for fine material in the channels is possible; a lower value could be used to ensure that the model does not try to represent mud-flow situations

Parameters used in every channel link

- threshold sediment concentrations in channels for infiltration and overbank sediment flow
- maximum thickness of top (active) bed sediment layer in channel; for gravel-bed channels typically one or two times $D_{99}$, the particle size for which 99% of the sediment is finer; for sand-bed channels the value might be of the order of 10 mm
- relationship between critical shear stresses for deposition and initiation of motion for fine sediment $\alpha < 1$; a value of 0.3 has been measured for coarse (gravel) sediment; likely range 0.25 - 0.75.

For each channel link

- initial bed sediment particle size distribution (on the same scale as the soil particle size distribution)
- type of soil on banks of each channel link;
- porosity of bed sediment; typical values 0.3 - 0.35 for coarse sand, 0.2 - 0.3 for sand and gravel, 0.15 - 0.4 for gravel;
- initial thickness of channel bed material; typical range 0.5 - 5 m, but could be larger for big rivers. A value of zero should represent a non-erodible river bed

- boundary conditions

For each grid square

- ground and rock covers $C_g$, $C_r$; ($C_g + C_r \leq 1$)
- initial depth of loose soil; probably zero and unlikely to exceed a few centimetres
- initial loose sediment composition - probably the same size distribution as the soil
- boundary conditions

For each soil type

- soil particle size distribution; For each soil type the user specifies what fraction of the soil particles are to be represented by each diameter.
- fractional clay content by weight, $F_c$, defined for particles smaller than 0.002 mm
- soil erodibility coefficients, $k_r$, $k_f$; these require calibration and have been evaluated in the ranges $k_r = 0.1 - 70\ J^{-1}$ and $k_f = 0.5 - 20\ mg\,m^{-2}\,s^{-1}$
- channel bank erodibility coefficient, $k_b$; as $k_f$ for soil
- soil porosity is not provided by the user in the sediment input file because it is assumed to equal the saturated soil moisture content (typically 0.3 - 0.55) and is transferred from the hydrology component

For each vegetation type:

- leaf drip diameter, $d_l$; typically 5 mm (see table 4.2 Wicks, 1988).
- proportion of drainage that falls as leaf drip $L_d$ (see table 4.2 in Wicks, 1988, for some suggested values for crops)
- leaf drip fall distance, X; typically the height of the base of the vegetation canopy above ground level.

## 5 LIST OF SYMBOLS

| | | |
|---|---|---|
| a | (-) | coefficient in the Yalin sediment load equation |
| $a_1$ & $b_1$ | (-) | coefficients used in the Marshall-Palmer raindrop momentum distribution |
| $a_2$ & $b_2$ | (-) | coefficients used to find leaf drip fall momentum |
| $a_3$ & $b_3$ | (-) | coefficients for calculating shear stress from the Shields curve |
| $a_4$ & $b_4$ | (-) | coefficients used in calculating K assuming rectangular channel |
| A | ($m^2$) | flow cross sectional area in channel |
| $A_i'$ | (-) | initial motion parameter in Ackers-White sediment load equation |
| $A_1$ | ($m^2$) | area of ground over which rainfall momentum, M is averaged |
| B | (m) | effective channel width, or active bed width for which there is sediment transport |
| c | ($m^3m^{-3}$) | sediment concentration |
| $c_{df}$ | ($m\,m^{-1}$) | concentration of fine sediment in the channel bed bottom layer (ratio of volume of fine sediment particles to total bulk volume of sediment in the bottom layer) |
| $c_f$ | ($m^3\,m^{-3}$) | concentration of fine sediment carried by the channel flow (ratio of the fine sediment particle volume to water volume) |

| | | |
|---|---|---|
| $c_i$ | ($m\,m^{-1}$) | concentration of sediment particles in size group i carried by the flow (ratio of the sediment particle volume to water volume) |
| $C_g$ | (-) | proportion of ground shielded by ground cover |
| $C_r$ | (-) | proportion of ground shielded by rock cover |
| $C_c$ | (-) | proportion of ground covered by vegetation canopy |
| CLAI | (-) | canopy leaf area index |
| $d_c$ | (m) | total thickness of the channel top bed sediment layer (all the sediment size groups) |
| $d_{c,i}$ | (m) | notional depth of sediment in channel top bed sediment layer in size group i |
| $d'_{c,i}$ | (-) | depth of sediment following any movement of sediment between the channel bed and the flow |
| $d_d$ | (m) | the depth of the channel bottom bed sediment layer |
| $d_l$ | (m) | leaf drip diameter (m) |
| $d_m$ | (m) | effective drip/drop diameter |
| $d_{min}$ | (m) | the minimum value allowed for $d_m$ (set to $10^{-4}$ m) |
| $D_a$ | (m) | critical diameter (the particle size in a nonuniform channel bed material which would begin to move at the same flow conditions as would a uniform bed material of size $D_a$) |

| | | |
|---|---|---|
| $D_f$ | (m) | fine sediment particle size |
| $D_{gra}$ | (m) | dimensionless sediment diameter for the critical diameter, $D_a$ |
| $D_i$ | (m) | representative sediment particle diameter for the size group i |
| DLS | (m) | total depth of loose sediment on the hillslope or bed sediment in the channel |
| DLSMAX | (m) | critical depth of loose sediment above which no further soil erosion may occur |
| $D_q$ | ($kgm^{-2}s^{-1}$) | rate of detachment of soil per unit area by overland flow |
| $D_r$ | ($kgm^{-2}s^{-1}$) | rate of detachment of soil by raindrop and leaf drip erosion |
| DRAINA | ($m s^{-1}$) | water drainage rate from canopy |
| $D_x$ | (m) | sediment diameter for which x% of the sediment is finer |
| $E_b$ | ($kgm^{-2}s^{-1}$) | the rate of detachment of material per unit area of channel bank |
| $E_{df}$ | ($kgs^{-1}m^{-1}$) | rate of deposition of fine sediment per unit length of channel |
| $f_i$ | (-) | proportion of the non-fine sediment in size group i available for transport |
| $f_{dc}$ | (-) | fraction of fine material in channel top bed sediment layer which is held against mobilization. |
| $F_{gr,i}$ | (-) | particle mobility used in Ackers-White sediment load equation |

| | | |
|---|---|---|
| $F_w$ | (-) | water depth correction factor for raindrop and leaf drip impact erosion equation |
| $FBETA_i$ | (-) | fraction of bed sediment (in both top and bottom layer) that is in the fine sediment size group |
| FBICRIT | (-) | critical fraction of bed sediment in fine size group above which infiltration cannot take place |
| $F_c$ | (-) | fractional clay content of the soil |
| $FDEL_i$ | (-) | volumetric concentration of mobile sediment in the non-fine sediment size group i |
| FICRIT | (-) | critical concentration of suspended fine sediment in channel flow below which bed infiltration cannot take place |
| FPCRIT | (-) | (i) in the overland flow subcomponent: maximum sustainable total sediment concentration<br>(ii) in the channel flow subcomponent: maximum sustainable sediment concentration in each separate sediment size group |
| g | ($ms^{-2}$) | acceleration due to gravity |
| $g_x$ & $g_y$ | ($m^2s^{-1}$) | volumetric overland sediment transport rates per unit width in the x and y directions respectively |
| G | ($m^3s^{-1}$) | total sediment transport capacity |
| $G_{eff,i}$ | (-) | effective sediment transport capacity for each non-fine sediment size group accounting for the contribution of other size groups |

| | | |
|---|---|---|
| $G_i$ | ($m^3s^{-1}$) | volumetric sediment particle transport rate for the sediment size fraction i |
| $G_{gr,i}$ | (-) | dimensionless sediment transport rate for the sediment size fraction i in the Ackers-White sediment load equation |
| $G_{sum}$ | ($m^3s^{-1}$) | potential sediment discharge based on availability of sediment |
| $G_{tot}$ | ($m^3s^{-1}$) | capacity particulate flow rate for overland flow (including all sediment size groups) |
| h | (m) | surface water depth |
| H | (m) | channel flow depth |
| i | (-) | subscript representing sediment size fraction |
| I | ($mm h^{-1}$) | is the rainfall intensity averaged over area $A_1$ |
| $k_b$ | ($kgm^{-2}s^{-1}$) | the bank erodibility coefficient |
| $k_i$ | (-) | coefficient in Ackers-White sediment load equation |
| $k_f$ | ($kgm^{-2}s^{-1}$) | overland flow soil erodibility coefficient |
| $k_r$ | ($J^{-1}$) | raindrop impact soil erodibility coefficient |
| K | (-) | proportionality constant in relationship between mean bed and bank shear stresses for channel flow |
| l | (m) | width of the overland flow |

| | | |
|---|---|---|
| $m_i$ | (m) | exponent in Ackers-White sediment load equation |
| $L_d$ | (-) | proportion of drainage that falls as leaf drip |
| M | (kg) | average mass of leaf drips |
| $M_{av}$ | ($kg^2m^2s^{-3}$) | average raindrop momentum squared per unit time |
| $M_d$ | ($kg^2s^{-3}$) | momentum squared of leaf drip reaching the ground per unit time per unit area |
| $M_r$ | ($kg^2s^{-3}$) | momentum squared of raindrops reaching the ground per unit time per unit area |
| $n_i$ | (-) | transition exponent for sediment size group i in Ackers-White sediment load equation |
| PLAI | (-) | plant leaf area index |
| PNETTO | ($m\,s^{-1}$) | net precipitation rate including drainage from canopy |
| $q_{si}$ | ($m^3s^{-1}m^{-1}$) | sediment input to channel from bank erosion and overland flow supply per unit channel length |
| Q | ($m^3s^{-1}$) | overland flow water discharge |
| $R_*$ | (-) | particle Reynolds number |
| s | (-) | sediment specific gravity |
| S | ($m\,m^{-1}$) | water surface slope in the direction of flow |

| | | |
|---|---|---|
| $t$ | (s) | time |
| $U$ | ($m\,s^{-1}$) | mean water flow velocity |
| $U_{si}$ | ($ms^{-1}$) | speed of sediment particles in size group i in channel flow |
| $u_*$ | ($m\,s^{-1}$) | shear velocity of water flow in channel |
| $u_{*ci}$ | ($ms^{-1}$) | critical value of shear velocity for initiation of motion for size group i |
| $V_d$ | ($ms^{-1}$) | leaf drip fall speed |
| $VDSED_i$ | ($m^3$) | volume of sediment in sediment size group i available for transport |
| $W_f$ | ($ms^{-1}$) | fine sediment settling velocity in channel flow |
| $x$ | (m) | distance in x direction, (Cartesian coordinates) |
| $X$ | (m) | average leaf drip fall distance |
| $y$ | (m) | distance in y direction, (Cartesian coordinates) |
| $z$ | (m) | depth of loose soil |
| $\alpha$ | (-) | constant representing ratio of critical shear stress for deposition to that for initiation of motion for fine sediment in channel flow (less than or equal to 1) |
| $\beta$ | ($kg\,m^{-1}$) | friction constant in equation for leaf drip fall speed |

| | | |
|---|---|---|
| $\delta$ | (-) | term in the Yalin sediment load equation |
| $\delta_0$ | (m) | maximum depth of the top bed sediment layer |
| $\lambda$ | (-) | loose sediment porosity |
| $\rho_s$ | ($kg\,m^{-3}$) | sediment particle density |
| $\rho$ | ($kg\,m^{-3}$) | water density |
| $\tau$ | ($Nm^{-2}$) | shear stress on hillslope due to overland flow or shear stress on the channel bed due to channel flow |
| $\tau_b$ | ($Nm^{-2}$) | shear stress acting on the bank |
| $\tau_{bc}$ | ($Nm^{-2}$) | critical shear stress for initiation of motion of bank material |
| $\tau_{dc,f}$ | ($N\,m^{-2}$) | critical shear stress for deposition of fine sediment particles on the channel bed |
| $\tau_{ec}$ | ($Nm^{-2}$) | critical shear stress due to overland flow for initiation of soil erosion |
| $\tau_{ec,f}$ | ($N\,m^{-2}$) | critical shear stress for initiation of motion of fine sediment particles in the channel bed |
| $\nu$ | ($m^2s^{-1}$) | kinematic viscosity of water |
| $\phi$ | (-) | bed sediment porosity |

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